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Reaction–diffusion network for geometric multiscale high speed image processing

S. Binczak [a,*], T. Sliwa [b], S. Jacquir [a], J.M. Bilbault [a]

[a] LE2I CNRS UMR 5158, Aile des Sciences de l'Ingénieur, Université de Bourgogne, BP 47870, 21078 Dijon Cedex, France
[b] LE2I CNRS UMR 5158, Université de Bourgogne, Route des plaines de l'Yonne, BP 16, 89010 Auxerre, France

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ABSTRACT

In the framework of heavy mid-level processing for high speed imaging, a nonlinear bi-dimensional network is proposed, allowing the implementation of active curve algorithms. Usually this efficient type of algorithm is prohibitive for real-time image processing due to its calculus charge and the inadequate structure for the use of serial or parallel architectures. Another kind of implementation philosophy is proposed here, by considering the active curve generated by a propagation phenomenon inspired from biological modeling. A programmable nonlinear reaction–diffusion system is proposed under front control and technological constraints. Geometric multiscale processing is presented and this opens a discussion about electronic implementation.

1. Introduction

The background framework of this work consists of the development of processing systems and their electronic implementation for high speed image intelligent sensors. Nowadays, many image processing techniques, particularly most linear or probabilistic ones, are included in real-time systems, but many families of mid-level techniques still remain unusable for such time processing requirements.

For high speed imaging applications, only some families of simple low-level techniques can be used, other techniques being applied on recorded data. For high speed imaging treatments, the approach of artificial retinas consisting in locating processing at the pixel stage gives some good results [1–3]. However, a significant disadvantage lies with this approach: many treatments either remain very difficult to adapt to retina architectures or imply development of specific architectures. To avoid this disadvantage, a generic type of image processing is proposed.

Mathematical morphology [4,5] has been specifically developed to meet image processing expectations. It helps to solve a major part of image processing problems and is widely used. High speed electronic implementations of basic morphological operators as dilations have already been studied, for instance with the use of a nonlinear electronic network [6]. However, the most important families of mathematical morphology treatments are based on two other main heavy operators which aregeodesic reconstruction and watershed algorithm.

Different architectures have been built in order to implement these mid-level treatments [7–9], most of them using numerical technologies, able to do real-time image processing for usual speeds, but with a technological gap preventing high speed imaging applications.

In this context, a possible generalization of the geodesic dilation is explored by constraining its construction with compatibility regarding a second connexity notion linked to the regularity of the shape of the objects. It is not expressed algebraically, but through a more intuitive approach. In fact, geodesic dilation can be interpreted as a topological wave propagation starting from a marker whose evolution is defined by the structure of the image. Partial Differential Equations (PDE) are a very convenient tool to generate this propagation where features of the image are included as intrinsic parameters of the equations. Furthermore, some of the reaction–diffusion PDE include a notion of regularity for the propagation phenomenon. For some of them, it is even possible to adjust parameters defining the capacity of the wave to travel through straight zones, holes, breaks and so on. Therefore, it defines a kind of geodesic-active curve or region.

Another aspect is very important for the notion of objects' regularity: the scale from which the object is observed. But carrying out multiscale analysis through successive multiple wave propagation is contrary to the high speed imaging technological gap. In order to reduce treatment times, the ideal case would be to achieve a multiscale analysis with only a one-pass wave propagation. In fact, a phenomenon producing propagation of wavefronts with increasing regularity along the height of the transition part is studied.

A systematic way of exploring dynamical systems described by PDE has been obtained by developing electronic circuits, which are very efficient for real-time analysis. These techniques lead to

* Corresponding author. Tel.: +33 380 396 848; fax: +33 380 395 910.
E-mail address: stbinc@u-bourgogne.fr (S. Binczak).

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analog computers, electrical lattices [10] or to the more general concept of Cellular Neural Network [11] (CNN). Although power computing has experienced exponential growth, these dedicated systems have benefited from integrated electronics and still remain an alternative for high speed processing. Thus, it is possible to use this kind of electronic device to obtain propagation speeds allowing it to overcome the high speed imaging technological gap.

Therefore, the purpose of this study is to propose a reaction–diffusion system with interesting properties, defined under technological constraints in order to suggest an electronic implementation. This article is organized as follows: a choice of reaction–diffusion system is presented in Section 2; then, the process of propagation of topological waves is studied according to the distribution of the local diffusive parameters. This provides a way to control the propagation paths of the travelling waves and to deduce a generic image processing method. In Section 3, this image processing technique is applied on specific images, enabling edge restoration/interpolation or one-pass multiscale approximation. This section is concluded by a preliminary result on grey level based segmentation, indicating that this system can lead to generic processing. Section 4 is devoted to open discussion on some possible implementation of an electronic circuit which could be able to perform high speed image processing. Finally, Section 5 concludes and gives perspectives of this study.

2. A discrete bistable system as a multiscale regularity PDE

Not all interesting reaction–diffusion PDE have realistic electronic implementations. When dealing with regular lattice, the simplest way to connect nodes leads to the classical heat equation. Unfortunately, in this medium, no stable propagation waves can exist. In order to overcome this limitation, a possible modification is to use the most general reaction–diffusion equation:

$$\frac{dv_{n,m}}{dt} = D_{n,m}[v_{n-1,m} + v_{n+1,m} + v_{n,m-1} + v_{n,m+1} - 4v_{n,m}] - f(v_{n,m}). \tag{1}$$

From a dynamical system analysis, the minimum number of equilibrium points is three in order to ensure the existence of stable propagating front wave with controllable shape. This naturally leads to the bistable equation.

This section presents a study of the propagation in a system modeled by this equation. It will be shown that, although this model is quite simple and its analog electronic implementation easily achievable, it allows the paths of propagating waves to be controlled, and therefore leads to deduce some interesting properties for image processing.

Let us consider a bi-dimensional regular discrete $(N,M) - length$ grid Ω on which the following bistable diffusive system is defined:

$$\begin{aligned}\frac{dv_{n,m}}{dt} = {} & D_{n,m}[v_{n-1,m} + v_{n+1,m} + v_{n,m-1} + v_{n,m+1} - 4v_{n,m}] \\ & - v_{n,m}(a - v_{n,m})(1 - v_{n,m}),\end{aligned} \tag{2}$$

where $D_{n,m}$ is a local diffusion parameter, a a threshold parameter and $v_{n,m}$ corresponds to the information located at node (n,m) (for instance, the intensity value of the pixel (n,m)).

The system is completed by the Neumann conditions (zero-flux conditions) on the border $\partial\Omega$ of the definition domain Ω, so that

$$\frac{\partial v_{n,m}}{\partial \eta} = 0 \quad if \ (n,m) \in \partial\Omega, \tag{3}$$

where $\frac{\partial}{\partial \eta}$ denotes the outer normally derivative at the boundary.

In the following section, propagation phenomena emerging from this system are investigated, from which a generic image processing method is then deduced.

2.1. Wavefronts in the homogeneous grid case

For the sake of simplicity, let us first consider the case where the local diffusion parameter is a constant so that

$$D_{n,m} = D \ \forall (n,m) \in \Omega. \tag{4}$$

This particular system corresponds to a discrete version of the FitzHugh–Nagumo PDE (without recovery term) which was established to describe propagation phenomena in various biological systems. In a one-dimensional space, it allows the study of the electric propagation of the leading edge of an action potential, i.e. the neuronal information travelling along the membrane of the nerve fibers [12,13]. Its discrete form (discrete Laplacian) corresponds to the myelinated nerve fibers case. In a continuous bidimensional space, it is widely used to study wavefront propagation in myocardial tissues [14].

In the uncoupled case, *i.e.* when $D = 0$, $v_{n,m} = 0$ and $v_{n,m} = 1$, $\forall \{n,m\}$ are two attracting steady states, while $v_{n,m} = a$, $\forall \{n,m\}$ is an unstable equilibrium point of the system, acting as a threshold.

In case of strong coupling, *i.e.* when D is large, we expect that a travelling wave will propagate depending on the value of a with a constant speed so that if $a < 1/2$ ($a > 1/2$ resp.), the steady state $v = 1$ ($v = 0$ resp.) will propagate at the expense of the steady state $v = 0$ ($v = 1$ resp.). When $a = 1/2$, no propagation occurs [15].

The response of the system, especially the shape of a wavefront, depends not only on the parameters of the system, but also on the initial conditions.

Let a marker be defined as a particular family of initial conditions for which $v_{i,j}(t = 0) = 1$ for a set of (i,j) values, $v_{k,l}(t = 0) = 0$ otherwise. The symmetry of the marker will determine the kind of travelling waves and its spatial shape. Among them, the two main propagating structures are the planar and the circular wavefronts. The following results are illustrated using numerical simulations obtained using a 4th order Runge–Kutta algorithm.

2.1.1. Planar waves

They emerge from a rectangular marker, as illustrated in Figs. 1 and 2 where two planar wavefronts propagate in opposing directions.

The symmetry induced by this choice of marker allows us to reduce the system to a one-dimensional space problem, expressed by:

$$\frac{dv_n}{dt} = D[v_{n-1} + v_{n+1} - 2v_n] - v_n(v_n - a)(v_n - 1). \tag{5}$$

This system of equation has been widely investigated to study the neural propagation in myelinated nerve fibers. From these studies, some important results can allow us to characterize the process of propagation, although no explicit overall analytical expression of the wavefront is available.

In the first place, when the coupling between nodes is strong, the differential-difference system (5) can be written, using a continuum approximation, so that

$$\frac{\partial v}{\partial t} = D\Delta v - v(v-a)(v-1), \tag{6}$$

where Δ denotes the Laplacian operator. Using a travelling wave analysis [15], one can express the propagation wave profile according to the travelling coordinate $\xi = n - ut$ and initial conditions ξ_0, and the unique velocity u of this topological wave as a function of D and a, so that

$$v(\xi) = \frac{1}{2}\left[1 \mp \tanh\left[\frac{\xi - \xi_0}{\sqrt{8aD}}\right]\right], \tag{7}$$

$$u = \pm(1 - 2a)\sqrt{\frac{D}{2}}. \tag{8}$$