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Connected image processing with multivariate attributes: An unsupervised Markovian classification approach ☆

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ABSTRACT

This article presents a new approach for constructing connected operators for image processing and analysis. It relies on a hierarchical Markovian unsupervised algorithm in order to classify the nodes of the traditional Max-Tree. This approach enables to naturally handle multivariate attributes in a robust non-local way. The technique is demonstrated on several image analysis tasks: filtering, segmentation, and source detection, on astronomical and biomedical images. The obtained results show that the method is competitive despite its general formulation. This article provides also a new insight in the field of hierarchical Markovian image processing showing that morphological trees can advantageously replace traditional quadtrees.

1. Introduction

In image processing, connected operators are morphological operators that concentrate on the non deformation of edges. In binary images, as their name suggests, their very principle is to work on the connected components of an image, and the only allowed operation is the removal of such components. Connected operators take their roots in the notion of filters by reconstruction [1,2]. They have been studied in the context of binary image processing in [3,4] and their extension to gray-scale images appeared in [5–8]. In this case, we say that an operator is connected if it is connected at all thresholding levels of the image.

Gray-scale connected operators started to become popular when a powerful framework and an efficient algorithm were proposed in order to compute connected operators by Salembier et al. [9]. This framework relies on a hierarchical representation of the image called the *Max-Tree* (also commonly known as the *(connected) component tree*). This representation is a tree where each node corresponds to a connected component of a threshold of the image and the parent relation is given by the inclusion relation among the connected components. This representation can then be used in the following 4-steps process (see Fig. 1) to define connected operators: (1) compute the Max-Tree of the image, (2) equip each node of the tree with some relevant attributes (area, compactness, moments, entropy, ...), (3) select nodes in the tree according to their attribute values, and finally (4) reconstruct an image from the filtered tree. The last reconstruction step consists in assigning a new value to each pixel of the image using the content of the nodes selected during the previous step. Connected operators have since been applied to various types of images: document images [10–12], biomedical imaging [13–16], remote sensing [17,18], or astronomical imaging [19,20]. Although the primary aim of connected operators was to perform image filtering [9], they have now been implied in various image analysis tasks such as segmentation [8,21,22], retrieval [23], classification [24] or registration [25].

It is also noteworthy that several connected operators (morphological reconstruction, flooding, region growing, ...) can be formulated in the framework of the Image Foresting Transform (IFT) [26]. The IFT relies on an efficient and versatile formulation based on the classical shortest path problem and provides a unified framework for a wide variety of image operators. On the other hand it has also been observed that connected operators share deep links with the TV-L1 optimization scheme [27].

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1.1. Related works

Researches have been carried out in order to improve the capacity or the efficiency at each step of the method.

About the construction of the representation, a first category of works concentrates on the definition of the connected components: connections [28], second-generation connections [29,30,14,31], hyperconnections [12], or directed connections [32]. The relations among various definitions of connections have been studied in [33].

Other works proposed to replace the Max-Tree by other representations, generally designated by *morphological trees*: binary partition trees [34], hierarchies induced by constrained connectivities [35], hierarchies of minimum spanning forests [36], the tree of shapes [37,38], the connectivity tree [39], the alpha-tree [40], ... One can note that the relations among some of these representations have been studied in [41,42].

The use and definition of relevant node attributes have also been studied in various works. Among the proposed attributes, we find: area [43,44], dynamic [45], simplicity and entropy [9], various geometrical moments and tensors [13,21], multispectral volume and color [20], contrast [10], Mumford-Shah energy [46] and so on.

While most connected operators designed so far rely on a simple thresholding on the attribute values – *i.e.*, a node is selected if its attribute value is larger than a specified threshold – it has immediately been noted that such a purely local strategy tends to be sensitive to noise whenever the attributes is not increasing: *i.e.*, when the attribute value does not vary monotonically along a branch of the tree. In the seminal work of Salembier et al. [9] the authors proposed several regularization strategies that all rely on the idea that the selection process should realize a pruning of the tree. The three proposed strategies were the followings:

- max: a branch is pruned at its highest selected node;
- min: a branch is pruned at its lowest selected node;
- Viterbi: an energy criterion is optimized in order to determine the best pruning level.

Recently, Xu et al. proposed in [16,47] to apply another connected filter on the tree representation seen as an image whose adjacency relation is given by the parent–children relation in the tree, and whose pixel values are the attribute values. This approach provides a non local strategy to detect relevant changes in the attribute values.

Another difficulty of the node selection process is the handling of multiple or multivariate attributes. In [48] Urbach et al. proposed two strategies to handle multivariate attributes: the direct extension of the thresholding approach where a node is selected if each of its attributes is greater than a particular threshold (equivalent to a marginal ordering of the attribute space), or using a distance to a reference attribute vector where a node is selected if the distance from its attribute vector to a reference vector is lower than a given threshold (equivalent to a reduced ordering of the attribute space). In both cases, there is a need to define a vector ordering which is a non trivial problem. In [49], the authors proposed to learn the distance function used to compare vector attributes from the distribution of the attributes of the nodes of interest. They modeled this distribution as a multivariate Gaussian, then learn the parameters of the distribution using a ground truth, and finally select the nodes with a threshold on the Mahalanobis distance defined by the model parameters. Although this approach was a first attempt to introduce some learning on multivariate data in the construction of connected operators, the node selection process still remained purely local.

Finally, the reconstruction step has been initially studied in [9] which defines a *natural* reconstruction: a pixel is valued with the level of the smallest selected node that contains it. Urbach *et al.* proposed in [24] to alleviate the problem of non increasing attributes by modifying the reconstruction process with the *subtractive* rule. In this strategy, whenever a node is removed, the level of its descendants are lowered by the height of the deleted node. This strategy has been used for the production of shape-size pattern spectra. Finally, in [11] the authors proposed a reconstruction process based on the notion of hyperconnection that enables the preservation of texture details during the reconstruction process.

1.2. Contributions

In this article, we propose a novel approach in order to select nodes in the Max-Tree. Our idea is to perform an unsupervised Bayesian classification of the nodes using a hierarchical Markovian model. This approach provides several benefits:

- a natural handling of multiple or multivariate attributes through the use of multivariate probability density functions;
- a robust global process for the classification of the nodes;
- an unsupervised approach that learns the model from each image without any ground truth.

Hierarchical Markovian models exist for about two decades now [50]. The advantage of these models compared to traditional hidden Markov random fields [51,52] is the possibly of computing exact inference without iterative algorithms. They have since been extensively applied to quadtrees [53,54] or similar structures [55]. In the same way, Hidden Markov Chains (HMC) allow an exact inference based on Maximum of Posterior Marginal (MPM) criterion and both approaches on chains [56,57] and quadtrees [58] have been refined through pairwise or triplet models in the past decade. Indeed, Markovian chains share similar properties with Markovian quadtrees than can be seen as hierarchical Markov chains [59].

In the proposed scheme, we use the Max-Tree to define a hierarchical Markovian probabilistic model. The attribute values of the nodes serve as observations, and we aim at finding the most probable hidden label value of each node of the tree. The existing iterative unsupervised classification algorithms for quadtree-like structures remain adapted to this model.

We propose several experiments in order to demonstrate how this approach can be used in different image analysis tasks: shape

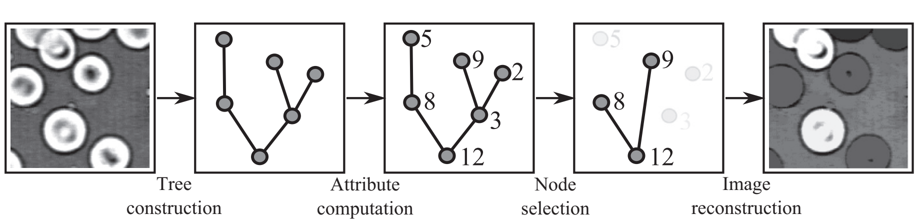

Fig. 1. The general 4 step process used to construct a connected operator: (1) construct the Max-Tree representing the image, (2) compute node attributes, *i.e.*, attach features to each node, (3) select nodes based on their attribute value: the selected nodes form a new tree, and (4) reconstruct an image from the new tree, *i.e.*, assign a value to each pixel based on the remaining nodes in the filtered tree.