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On energy-to-peak filtering for semi-Markov jump singular systems with unideal measurements☆

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ABSTRACT

This paper focuses on the energy-to-peak filtering issue for a class of singular semi-Markov jump systems with unideal measurements. Some network-induced phenomena, such as sensor nonlinearity and packet dropouts caused by the unideal measurements are considered. The occurrence of sensor nonlinearity is described in a random way and obeyed a Bernoulli distribution. In the framework of the Lyapunov–Krasovskii stability theory, some sufficient conditions are given to ensure that the considered error system is stochastically mean-square stable and guarantees an energy-to-peak (or called $L_2 - L_\infty$) performance level. On the basis of these conditions, an available design method to the desired filter is proposed drawing support from an improved matrix decoupling approach. For showing the effectiveness and superiority of the proposed method, we finally provide two illustrated examples.

1. Introduction

Singular systems (SSs) are regard as implicit systems, differential-algebraic systems, generalized systems, descriptor systems or semi-state systems. Owing to their approximate description of some physical systems compared with the state-space systems, SSs have been applied in quantities of areas, such as, power systems, electrical circuits, chemical processes and others. As a consequence, it is no wonder that SSs have attracted particular research attention during the last several decades and tremendous research progress has been generated, (see [1–4], and the references therein). Furthermore, it is certainly worth pointing out that a kind of stochastic SSs, i.e., Markov jump singular systems (MJSSs), has been seen as a highlight during the development of SSs [5]. In fact, MJSSs represent the stochastic switched systems composed of a number of sub-systems where switching among themselves is governed by a Markov chain. Because of the above light spot of Markov jump systems (MJSs) [6–9], MJSSs have the unique advantage for modelling the SSs with abrupt changes in their structures. Therefore, quite a few research efforts have been devoted to the study of MJSSs. For instance, some design methods to solving the H_∞ filtering and control issues for continuous-time MJSSs were proposed in [10] and [11], respectively. In the context of discrete-time MJSSs, the H_∞ filtering problem was addressed in [12].

Although the MJSSs get the favour of many researchers for their better ability to model systems, it still has a certain degree of boundedness in some scenarios due to the inherent imperfections of MJSs. A truism restriction for MJSs is that the sojourn-time existed in two consecutive jumps is required to follow the exponential distribution which is a memoryless distribution [13]. Such a requirement on MJSs is too harsh, which not only is unreasonable in some practical applications but also brings some conservative. So as to relax the restriction, a new class of MJSs called semi-Markov jump systems (SMJSs) [14–17] was proposed. Very recently, due to the fact that SMJSs have maintained momentum not only in practice but also in theory, quite a few efforts have been assembled into this topic [18,19]. Unfortunately, a drawback of the above-mentioned works is lack of considering the feature of singular systems in the study of SMJSs [20,21]. Fortunately, Wang *et. al.* originally investigated the study for continuous-time semi-Markov jumping singular systems (SMJSSs) with uncertainty and nonlinearity in [22], where a sufficient condition was presented to successfully ensure that the nonlinear SMJSSs were stochastically admissible. It should be also pointed out that the network induced

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phenomena were not fully considered in [22]. Such a regret provides the first motivation of our recent work. Different from [22], we focus on the issue of energy-to-peak filtering for SMJSSs, where the phenomena of sensor nonlinearity and packet dropouts occur simultaneously.

In parallel, the robust filtering problem is a permanent topic in the study of SSs with external noises. In the practical situation, some system states are often difficult to be measurable, or even unmeasurable due to technical constraints and other factors [23–30] . How to estimate the unmeasurable states becomes an interesting question. Mainly for the above reason, the concept of filtering has been put forward. With tremendous attention from researchers, the filtering technique has developed rapidly, from the conventional Kalman filtering which has long enjoyed a good reputation for handling the systems with white noise to energy-to-peak filtering and H_∞ filtering which are only required the noise signal is the ∞-norm and energy-bounded. More precisely, for the issue of energy-to-peak filtering, which includes of minimizing the peak error value, the filter design with ensured the fixed energy-to-peak performance of uncertain systems was discussed in [31,32]. The results in [33] coped perfectly with such a problem for a class of MJSSs. It is worth remarkable that the results in [31,33] were obtained based on an ideal network transmission environment, that is, any network induced phenomenon does not occur. However, such an assumption could be hardly guaranteed due to the complication of environments. Therefore, it is necessary to consider the filtering problem for systems with unreliable links. Above all, how to address this intriguing question is the another motivation of this work.

Summarizing the discussions mentioned above, this paper addresses the issue of energy-to-peak filtering for SMJSSs. A construction method of the mode-dependent filter is given to ensure the resulting error system is stochastically mean-square stable with an energy-to-peak disturbance rejection attenuation level. There are three main contributions in this paper: 1) The issue of energy-to-peak filtering for a class of quite comprehensive system models (i.e. SMJSSs) is investigated for the first time; 2) The unideal measurements case is fully taken into account, and accordingly the occurrence of some network induced phenomena include sensor nonlinearity and packet dropouts caused by the unideal measurements are described in a stochastic way. 3) An improved matrix decoupling approach, which may provide more free degree and flexibility to structure the desired filter than that in [10], is introduced.

2. Problem formulation

Considering the following class of linear continue-time singular system (Σ)

$$E\dot{x}(t) = A(\alpha(t))x(t) + B(\alpha(t))\omega(t), \tag{1}$$

$$y(t) = \delta(\alpha(t),t)C(\alpha(t))x(t) + (1-\delta(\alpha(t),t))\Psi(x(t)), \tag{2}$$

$$z(t) = D(\alpha(t))x(t), \tag{3}$$

where $x(t)\in\mathbb{R}^a$, $y(t)\in\mathbb{R}^b$ and $z(t)\in\mathbb{R}^c$ are the system state and the measurement output and the controlled output, respectively. $\omega(t)\in\mathbb{R}^d$ is the disturbance. $E\in\mathbb{R}^{a\times a}$ may be a singular matrix, and $A(\alpha(t))$, $B(\alpha(t))$, $C(\alpha(t))$ and $D(\alpha(t))$ are fixed real constant matrices with appropriate dimensions. The random variable $\{\alpha(t)\}$ stands for a semi-Markov jump process with right continuous trajectories, which is homogeneous, finite-state and takes discrete values in a fixed set $\mathcal{S}=\{1,2,\dots,r\}$. Furthermore, its transition probability matrix $\prod \triangleq \{\pi_{mn}(\triangle)\}$ is described by

$$\Pr\{\alpha(t+\triangle)=n|\alpha(t)=m\} = \begin{cases}\pi_{mn}(\triangle)\triangle + o(\triangle), & m\neq n\\ 1+\pi_{mn}\triangle + o(\triangle), & m=n\end{cases}, \tag{4}$$

where $\triangle>0$ is the sojourn time, $\lim_{\triangle\to 0}(o(\triangle)/\triangle)=0$ and $\pi_{mn}(\triangle)\geq 0$, for $n\neq m$, is the transition rate from mode m at time t to mode n at time $t+\triangle$ and

$$\pi_{mm}(\triangle) = -\sum_{n\in\mathcal{S},n\neq m}\pi_{mn}(\triangle).$$

Remark 1. As stated in [18], the transition rate of semi-Markov jump process is often partly available. Therefore, we assume that the transition rate $\pi_{mn}(\triangle)$ is in the range of $\left[\pi_{mn}^i,\pi_{mn}^j\right]$ and can be naturally rewritten as follows

$$\pi_{mn}(\triangle) = \sum_{h=1}^{\mathcal{H}}\beta_h\pi_{mn,h},\ \sum_{h=1}^{\mathcal{H}}\beta_h = 1,\ \beta_h\geqslant 0, \tag{5}$$

and

$$\pi_{mn,h} = \begin{cases}\pi_{mn,h}^i + (h-1)\frac{\pi_{mn,h}^j-\pi_{mn,h}^i}{\mathcal{H}-1}, & n\neq m, \quad n\in\mathcal{S}\\ \pi_{mn,h}^j - (h-1)\frac{\pi_{mn,h}^j-\pi_{mn,h}^i}{\mathcal{H}-1}, & n=m, \quad n\in\mathcal{S}\end{cases}. \tag{6}$$

Specifically, the nonlinear function $\Psi(\cdot)$ is assumed to satisfy the following condition for each $x,y\in\mathbb{R}^a$ [34]

$$[\Psi(x)-\Psi(y)-Y_1(x-y)]^T[\Psi(x)-\Psi(y)-Y_2(x-y)]\leqslant 0,\ \Psi(0)=0, \tag{7}$$

where Y_1 and Y_2 are appropriate dimensional constant matrices known beforehand. $\delta(\alpha(t),\,t)$ are mode-dependent random variables taking values 0 or 1. They obey the following probability distribution laws

$$\Pr\{\delta(\alpha(t),t)=1\} = \mathcal{E}\{\delta(\alpha(t),t)\} = \bar{\delta}(\alpha(t)),$$
$$\Pr\{\delta(\alpha(t),t)=0\} = 1-\bar{\delta}(\alpha(t)),$$

where $\bar{\delta}(\alpha(t))\in[0,1]$ are known beforehand constants. Clearly, for the stochastic variables $\delta(\alpha(t),\,t)$, one has

$$\mathcal{E}\left\{\delta(\alpha(t),t)-\bar{\delta}(\alpha(t))\right\} = 0,$$
$$\mathcal{E}\left\{|\delta(\alpha(t),t)-\bar{\delta}(\alpha(t))|^2\right\} = \bar{\delta}(\alpha(t))\left(1-\bar{\delta}(\alpha(t))\right). \tag{8}$$

Remark 2. For modeling the randomly occurring of sensor nonlinearity and packet dropouts, the mode-dependent random variables $\delta(\alpha(t),\,t)$ are taken into account, which are obeyed the Bernoulli distribution. Form the description of measurement output $y(t)$ in (2), we can easily discover the following two facts:

1) under the condition of $\delta(\alpha(t),t)=0$, the equality of (2) will degrades to $y(t)=\Psi(x(t))$, which only has the sensor nonlinear and denotes the packet dropout;

2) under the condition of $\delta(\alpha(t),t)=1$, the equality of (2) will changes to $y(t)=C(\alpha(t))x(t)$, which only has the normal measurement output and expresses the normal case.

In this paper, we are interested in designing a filter as follow

$$E\dot{x}_f(t) = A_f(\alpha(t))x_f(t) + B_f(\alpha(t))y(t), \tag{9}$$

$$z_f(t) = C_f(\alpha(t))x_f(t), \tag{10}$$

where $x_f(t)$ is the filter state vector, $z_f(t)$ is the estimate of $z(t)$, $A_f(\alpha(t))$, $B_f(\alpha(t))$ and $C_f(\alpha(t))$ are constant real matrices of filter to be determined. From convenience point of view, we denote $A_m = A(\alpha(t))$ and $A_{fm} = A_f(\alpha(t))$ for each $\alpha(t)=m\in\mathcal{S}$, and the other symbols are similar denoted.