JID:IJA AID:8054 /FLA [m3G; v1.215; Prn:11/05/2017; 15:58] P.1 (1-27)

International Journal of Approximate Reasoning ••• (••••) •••–•••

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Generalizing type-2 fuzzy ontologies and type-2 fuzzy description logics ☆

Fernando Bobillo [a], Umberto Straccia [b]

[a] Aragon Institute of Engineering Research (I3A), University of Zaragoza, Spain
[b] Istituto di Scienza e Tecnologie dell'Informazione, Consiglio Nazionale delle Ricerche (ISTI-CNR), Pisa, Italy

ARTICLE INFO

Article history:
Received 22 February 2016
Received in revised form 21 April 2017
Accepted 30 April 2017
Available online xxxx

Keywords:
Fuzzy ontologies
Fuzzy description logics
Type-2 fuzzy sets
Logic for the semantic web

ABSTRACT

In the last years, we are witnessing an increase of real-world applications of fuzzy ontologies. Most fuzzy ontologies are based on type-1 fuzzy logic, and type-2 fuzzy ontologies have not yet received such attention so far. Furthermore, there exists an important gap between type-2 knowledge representation formalisms (type-2 Description Logics) and type-2 fuzzy ontology applications. In this paper, we propose a formal framework for type-2 fuzzy ontologies taking into account the needs of existing applications. Essentially, our approach makes it possible to manage some uncertainty in the fuzzy membership functions used in the fuzzy datatypes and in the degrees of truth of the axioms. We define a type-2 Description Logic, a reasoning algorithm, and give a Fuzzy OWL 2 specification of it.

1. Introduction

In the last decade, *OWL ontologies* have become standard for knowledge representation. An ontology is an explicit and formal specification of the concepts, individuals and relationships that exist in some area of interest, created by defining axioms that describe the properties of these entities [52]. Ontologies can provide semantics to data, making knowledge maintenance, information integration, and reuse of components easier.

The theoretical underpinnings of ontologies are strongly based on *Description Logics* (DLs) [4]. DLs are a family of logics for representing structured knowledge that play a key role in the design of ontologies. Notably, DLs are essential in the design of OWL 2 (Web Ontology Language) [26], the current standard language to represent ontologies. As a matter of fact, OWL 2 is almost equivalent to the DL $\mathcal{SROIQ}(\mathbf{D})$ [33].

Despite the undisputed success of ontologies, it has been widely pointed out that classical ontologies are not appropriate to deal with imprecise and vague knowledge, inherent to several real world domains [56]. Fuzzy set theory and fuzzy logic [60] have proved to be suitable formalisms to handle these types of knowledge. Therefore *fuzzy ontologies* emerge as useful in several applications, such as [7,28,29,43,50,57].

Most of the times, fuzzy ontologies are based on the notion of type-1 fuzzy set. That is, while in classical set theory elements either belong to a set or not, elements can belong to a type-1 fuzzy set to some degree. Thus, fuzzy sets can be characterized by a membership function assigning to every element a degree of truth.

☆ This paper is part of the virtual special issue on Advances in Weighted Logics for Artificial Intelligence, edited by Marcelo Finger, Lluis Godo, Henri Prade and Guilin Qi.
E-mail addresses: fbobillo@unizar.es (F. Bobillo), straccia@isti.cnr.it (U. Straccia).

http://dx.doi.org/10.1016/j.ijar.2017.04.012
0888-613X/© 2017 Elsevier Inc. All rights reserved.

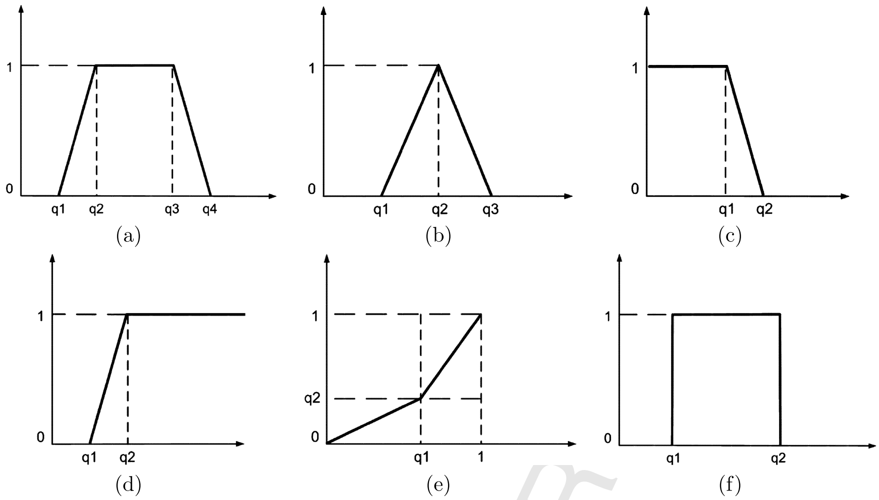

Fig. 1. (a) Trapezoidal function; (b) Triangular function; (c) Left-shoulder function; (d) Right shoulder function; (e) Linear function; (f) Crisp interval.

A limitation of these type-1 fuzzy sets is that they are based on the existence of well-known membership functions, with no uncertainty associated to them. However, in practice, it is often difficult to precisely know the membership function of a fuzzy set. A possible solution is to consider membership functions that are themselves fuzzy sets, yielding the so-called type-2 fuzzy sets [61]. It has been demonstrated in practice that type-2 fuzzy sets usually handle the imprecision related to the definition of fuzzy memberships in a better way.

Some type-2 fuzzy ontologies and type-2 fuzzy DLs have been proposed in the literature (see Section 3 for details), but current works are at a preliminary stage. On the one hand, the previously proposed type-2 fuzzy DLs are not expressive enough to cover the needs of type-2 fuzzy ontology applications presented so far. Moreover, current type-2 fuzzy ontology applications are not based on fuzzy DLs, so it is not possible to take advantage of automatic reasoning services, and developers must implement their own inference strategies.

In summary, in this paper a formal framework for type-2 fuzzy ontologies is proposed by taking into account the needs of existing applications. In particular, we define a type-2 fuzzy DL and provide a reasoning algorithm. Furthermore, we will discuss how to integrate this extension into existing type-1 fuzzy ontology languages and clarify the relation between type-1 fuzzy DLs and interval fuzzy DLs.

The remainder of this paper is organized as follows. Section 2 starts by providing some background on type-1 and type-2 fuzzy sets that will be needed to follow this paper. Section 3 performs a critical overview of the related work on type-2 ontologies and DLs. Then, we introduce type-1 and type-2 DLs in Section 4. Next, in Section 5 we state some results showing the lack of expressivity of previous works on type-2 DLs. After that, Section 6 discusses how to represent type-2 ontologies using fuzzy ontology languages and Section 7 provides a reasoning algorithm for general type-2 DLs. Finally, Section 8 sets out some conclusions and ideas for future research.

2. Preliminaries

Type-1 fuzzy logic. Fuzzy set theory and fuzzy logic were proposed by L.A. Zadeh [60] to manage imprecise and vague knowledge. While in classical set theory elements either belong to a set or not, in fuzzy set theory elements can belong to some degree. More formally, let X be a set of elements called the reference set. A *type-1 fuzzy subset* A of X is defined by a membership function $\mu_A(x)$, or simply $A(x)$, which assigns to every $x \in X$ a degree of truth, measured as a value in a truth space $\mathcal{N}$. The truth space is usually $\mathcal{N} = [0, 1]$, but other choices are possible. Indeed, $\mathcal{N}$ does not need to be a total order, nor does it need to be infinite. As in the classical case, 0 means no-membership and 1 full membership, but now a value between 0 and 1 represents the extent to which x can be considered as an element of the fuzzy set A.

Some popular membership functions, commonly used to define fuzzy sets, are the trapezoidal (Fig. 1(a)), the triangular (Fig. 1(b)), the left-shoulder (Fig. 1(c)), the right-shoulder (Fig. 1(d)), and the linear function (Fig. 1(e)).

Fuzzy logics provide compositional calculi of degrees of truth. The conjunction, disjunction, complement and implication operations are performed in the fuzzy case by a *t-norm* function $\otimes$, a *t-conorm* function $\oplus$, a *negation* function $\ominus$ and an *implication* function $\Rightarrow$, respectively. For a formal definition of these functions we refer the reader to [31,36].

A quadruple composed by a t-norm, a t-conorm, an implication function and a negation function determines a *fuzzy logic*. One usually distinguishes three fuzzy logics, namely Łukasiewicz, Gödel, and Product [31], due to the fact that any continuous t-norm can be obtained as a combination of Łukasiewicz, Gödel, and Product t-norms [49]. It is also usual to consider Zadeh logic, including the conjunction, disjunction, and negation originally proposed by Zadeh [60] together with Kleene–Dienes implication defined as $\alpha \Rightarrow_{KD} \beta = \max(1 - \alpha, \beta)$. The name of Zadeh fuzzy logic is used following the

Please cite this article in press as: F. Bobillo, U. Straccia, Generalizing type-2 fuzzy ontologies and type-2 fuzzy description logics, Int. J. Approx. Reason. (2017), http://dx.doi.org/10.1016/j.ijar.2017.04.012