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Fuzzy-logic control of an inverted pendulum on a cart[☆]

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ABSTRACT

The inverted pendulum on a cart is an under actuated, unstable non-linear system that is used as a benchmarking problems in control theory. The non-linear nature of the system makes linear controllers, such as the proportional integral derivative (PID) controllers, possibly unfeasible as they only guarantee stability of a linear system. A fuzzy-logic controller provides many different stable controllers applicable to inverted pendulum on a cart. In this paper, the fuzzy parallel distributed compensation (PDC) controller is introduced and implemented on an unstable system, and the performance is demonstrated in MATLAB Simulink. The fuzzy PDC controller is dependent on the Takagi–Sugeno (TS) fuzzy model to obtain the state feedback gains required by solving the linear matrix inequalities (LMI). The LMI produced satisfactory results for all initial pendulum positions simulated even under uniformed disturbance. Our results have been compared against two other works to reveal the effectiveness of the proposed model.

1. Introduction

An inverted pendulum on a cart presents a classical problem in dynamics and control theory due to its complex, multi-variable, non-linear, and unstable system. It consists of two degrees of freedom (DOF), an inverted pendulum which is attached to a cart, where the only actuated DOF provides the horizontal motion of the cart. This means that for all control algorithms to be tested on this system, the main objective is to stabilize the pendulum in the upright position for any initial pendulum position above the horizontal axis. As it is a benchmarking problem for testing control algorithms, numerous research has been reported encompassing Proportional Integral Derivative (PID) controller [1], Linear Quadratic Regulators (LQR) [1], state-space controller [2], Neural Networks (NN) [3], and fuzzy-logic controllers [4–11].

Applications range from simulating the human balancing system [12], missile guidance systems, seismometers and most notably, self-balancing two-wheeled vehicle such as the one which has already been commercialized as the Segway Personal Transporter (PT) [13].

The term ‘fuzzy’ implies that decisions made by a machine cannot be exclusively expressed as ‘true’ or ‘false’, but instead as partially ‘true’ and ‘false’ for each of the defined linguistic variables such as ‘high’, ‘medium’, ‘low’ . These partial ‘truths’ are expressed as membership functions ranging from zero to one quantifying how true a linguistic variable is. All of the linguistic variables and their membership functions are used in the rule- base design to determine the fuzzy controller output.

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Fuzzy-logic control provides a natural, flexible and intuitive way to express ambiguous responses in systems where the dynamics may be too complex or not known. Fuzzy-logic control can also be combined with conventional controllers such as the linear PID controller, which traditionally only guarantees stability for linear systems, allowing it to control a non-linear system, hence providing improved robustness and response as demonstrated in [5,8].

The use of the Fuzzy PID controller to control an inverted pendulum on a cart was discussed in [4,5]. While [4] implemented a fuzzy PD controller to stabilize the system, an LQR and a conventional PID controller are implemented in [5] to compare the performance with the fuzzy-logic controller. The performance showed that the fuzzy-logic controller provided much better response or settling time, lowest overshoot, and the least fluctuations.

The idea behind the controller is to design a rule-base such that mimics the ideal response of the conventional PID controllers. For this purpose, equations governing conventional PID controllers are discretized to arrive at the fuzzy model, where the output of the controller depends on the error-prone defuzzification process and the derivative of the system as the inputs. This type of controller uses the Mamdani based Fuzzy Inference Systems (FIS) for the construction of the rule base and the defuzzification process.

Commonly, the derivative, integral, and proportional gains are usually optimized via LQR method in [6], this has been further expanded by fine-tuning the gains of the controller using an immune PD control algorithm. The algorithm works by attempting to mimic the immune mechanism in a biological system. The implementation, the fuzzy immune PD controller yields improved settling time and reduced overshoot when compared to the LQR tuned fuzzy PD controller.

For the PDC controller, the Takagi–Sugeno FIS is used. Initially introduced in [8] and emulated in [9], the controller is applied on a simplified dynamic model of the inverted pendulum on a cart. The simplification is achieved by neglecting the Lagrange cart position equations, since the problem only involves the stabilization of the inverted pendulum, not the control of the cart. In our paper, local approximation is used to create the rule-base of the TS fuzzy model. Results show that the controller is stabile to the inverted pendulum at all positions when compared to the basic linear controller which only allowed stabilization for angles ranging from $-\frac{\pi}{4}$ to $\frac{\pi}{4}r$.

However, it is mentioned in [9] that for some systems, the local approximation method does not guarantee stabilization. From this, [7] attempted the full dynamic model, and the sector non-linearity instead of local approximation to construct the rule-base for the design of the controller. For three points of initial pendulum position and their derivatives tested, the pendulum achieved less than ideal stabilization containing fluctuations. The use of sector non-linearity guarantees stabilization of the controller with the expense of design feasibility. A performance-oriented PDC was also initially introduced and implemented in [18], which works on the assumption that the compensating gains for the conventional fuzzy PDC controller is not fixed. The performance-oriented PDC showed better results when compared to the conventional fuzzy PDC method.

In [10], the use of a robust fuzzy controller is to compensate the approximation error that causes stabilization issues due to the use of local approximation in the construction of the rule-base. The robust fuzzy controller is simply an improvement to the fuzzy PDC controller, allowing it to guarantee stabilization of the controller, while reducing the work-load on the design of the rule-base. Results showed satisfactory performance where the performance was better than the conventional fuzzy PDC controller.

The optimal fuzzy controller is designed to optimize the fuzzy PDC controller or the robust fuzzy controller providing better response and less overshoot as demonstrated in [9]. Reference [11] implemented this controller on the inverted pendulum on a cart system with decent performances.

The paper is structured as follows: in Section 2, the PDC controller has been built and the Takagi-Sugeno has been introduced and explained. In Section 3, the dynamics of inverted pendulum on a cart has been explained, and the dynamics equations have been derived. The fuzzy controller has been designed in Section 4. Section 5 showed the simulation and results of the controller and the comparison against other works. Finally, Section 6 concludes the results and findings of the paper.

2. Fuzzy Inference Systems

In this section, the Takagi–Sugeno is selected and explained. Next, the PDC controller is built based on the TS fuzzy model and the structure of the controller is covered. The PID controller is also briefly explained since it will be used for comparison against the fuzzy controller.

The output of the controller for any fuzzy controller is governed by the defuzzification process and its rule-base after which the inputs are fuzzified into membership functions. The defuzzification process depends on two FIS: Mamdani and Takagi–Sugeno. Fig. 1 describes the fuzzy inference system.

While both FIS uses membership functions to fuzzify the inputs or premise variables, Takagi–Sugeno FIS uses a weighted average method to compute the output of the fuzzy model; which is different from the Mamdani FIS that requires the defuzzification of the consequent to compute the output. In other words, the consequent for Mamdani FIS rule-base is fuzzy in nature, while the consequent for Takagi–Sugeno FIS is not. This fuzzy nature allows Mamdani FIS to be more intuitive and interpretable in design at the expense of computational power. The lack of a fuzzy consequent allows the integration of Takagi–Sugeno FIS with adaptive and optimization techniques. For these reasons, the Takagi–Sugeno FIS is selected in this paper. The difference in the defuzzification process is depicted in Fig. 2.