International Review of Financial Analysis 53 (2017) 80–93

Contents lists available at ScienceDirect

International Review of Financial Analysis

Parameter estimation risk in asset pricing and risk management: A Bayesian approach

Radu Tunaru[a,b,*], Teng Zheng[a,b]

[a] CEQUFIN, Kent Business School, University of Kent, UK
[b] Kent Business School, Canterbury, Kent CT2 7PE, UK

ARTICLE INFO

Article history:
Received 13 July 2016
Received in revised form 7 June 2017
Accepted 22 August 2017
Available online 25 August 2017

JEL classification:
G13
G17
G32
C53

Keywords:
Parameter estimation risk
Bayesian option pricing
Greeks
Probability of default

ABSTRACT

Parameter estimation risk is non-trivial in both asset pricing and risk management. We adopt a Bayesian estimation paradigm supported by the Markov Chain Monte Carlo inferential techniques to incorporate parameter estimation risk in financial modelling. In option pricing activities, we find that the Merton's Jump-Diffusion (MJD) model outperforms the Black-Scholes (BS) model both in-sample and out-of-sample. In addition, the construction of Bayesian posterior option price distributions under the two well-known models offers a robust view to the influence of parameter estimation risk on option prices as well as other quantities of interest in finance such as probabilities of default. We derive a VaR-type parameter estimation risk measure for option pricing and we show that parameter estimation risk can bring significant impact to Greeks' hedging activities. Regarding the computation of default probabilities, we find that the impact of parameter estimation risk increases with gearing level, and could alter important risk management decisions.

1. Introduction

Since the financial crisis of 2008, model risk has attracted more attention in academic research. Andrikopoulos (2015) argues that the "true" asset value in the financial world is quite often model dependent. Portfolio selection, trading strategies and corporate finance decisions are consequentially made based on these "true" values as well as the difference between the market value and the "true" value. Therefore, risk that stems from financial modelling can have a substantial impact on financial quantities such as option prices, hedging ratios or default probabilities.

Model risk was linked to a long series of significant events in the financial markets, see Jacque (2015). In 1987, Merrill Lynch reported losses of $300 million on stripped mortgage-backed securities caused by an incorrect pricing model. In 1992, J.P. Morgan lost about $200 million in the mortgage-backed securities market due to the inadequate modelling of prepayments. Later in 1997, the New York subsidiary of the Bank of Tokyo/Mitsubishi lost $83 million because their internal pricing model overvalued OTM Bermudan swaptions (Dowd, 2002). A Deutsche Bank subsidiary in Japan traded electronically using some smart models which went out of control in June 2010. One model went into an infinite loop and took out a $183 billion stock position (Tunaru, 2015). More recently in 2013, J.P. Morgan revealed a trading loss of more than $6.2 billion, which was indirectly caused by the underestimation of risk level by their value-at-risk (VaR) model. Model risk is also identified by the Basel Committee on Banking Supervision in the old and new Basel frameworks; see Basel Committee on Banking Supervision (2006, 2011), Detering and Packham (2016). Financial institutions must gauge their model risk to fulfil regulatory requirements (Kerkhof, Melenberg, & Schumacher, 2010).

Focused on parameter estimation risk, in this study, we adopt a Bayesian approach to financial modelling with applications to option pricing, including hedging ratios, and to default probabilities calculations. We advocate using distributions of any quantity of interest to the investor. The distributions are generated by the Markov Chain Monte Carlo (MCMC) inferential process. Existing literature eloquently explains how to implement the Bayesian estimation framework to option pricing models, yet empirical applications are

* Corresponding author.
E-mail addresses: R.Tunaru@kent.ac.uk (R. Tunaru), T.Zheng@kent.ac.uk (T. Zheng).

http://dx.doi.org/10.1016/j.irfa.2017.08.004
9999-9994//

very limited. We advocate an improved methodology for investigating the impact of parameter estimation risk towards option pricing, hedging and risk management activities. Moreover, we propose a VaR-type technique to measure parameter estimation risk in option pricing. Interestingly, our results indicate that model risk may not be symmetric for the buyer and the seller in a derivative contract.

Parameter estimation risk is an important source of model risk (Glasserman & Xu, 2014; Tunaru, 2015). It refers to the uncertainty of estimating the correct parameter values given a model structure. Standard practices in financial modelling are based on point estimations of parameters, with standard estimation errors being ignored completely in both asset pricing and risk management computations. Any estimation method would induce a certain level of parameter estimation risk. There is no justification for neglecting any of these errors.

While most market makers and researchers agree that liquidly traded option prices of major stocks and indices are determined by market supply and demand, less liquid options such as exotic options do not have available market prices and depend heavily on models to determine their values (Cont, 2006; Jacquier & Jarrow, 2000; Dahlbokum, 2010). Therefore, parameter estimation risk in option pricing is of great interest to many market participants. Jacquier and Jarrow (2000) carried out a study to incorporate parameter estimation risk of the Black-Scholes (BS) model and its non-parametric extensions into option pricing using the Bayesian estimation approach. They found that even upon consideration of parameter estimation risk, these models cannot deliver promising results in forecasting due to rigid model assumptions. They suggest that further study should be extended to use models with parameters capturing missing time varying dynamics (e.g. jump process). Later studies also confirm the capability of Bayesian econometrics in capturing model uncertainty as well as the feasibility of implementing it in financial practices, but most of this literature focuses on describing the methodology; see for example Bunnin, Guo, and Ren (2002), Jacquier and Polson (2010), Johannes and Polson (2010). Other contributions to the literature emphasise the advantage of extracting latent parameters using the Bayesian estimation approach, but paying little attention to its application in dealing with parameter estimation risk in practices; see for example Eraker, Johannes, and Polson (2003), Yu, Li, and Wells (2011), Kaeck and Alexander (2013).

The Bayesian method advances in its ability of delivering the joint posterior distribution of parameters, which contains all possible value of the parameters given the model and the observed data, so that shapes of the distributions as well as credibility intervals can be obtained easily (Laurini & Hotta, 2010). Therefore, under the Bayesian framework, all parameters are stochastic, accounting for the uncertainty in their estimation. We highlight our proposed methodology using two well-known models as vehicles of research, the Black-Scholes (BS) model and the Merton Jump-Diffusion model (MJD). The MJD model was developed by Merton (1976) as a key alternative model to the BS model, capable of generating kurtosis and skewness in line with empirical literature on stock returns; see Bakshi, Cao, and Chen (1997) and Dahlbokum (2010). Nevertheless, this model has been omitted from most of the related literature which provides empirical tests (Jacquier & Jarrow, 2000; Eraker et al., 2003; Gupta & Reisinger, 2014; Kaeck & Alexander, 2013; Yu et al., 2011), except for Frey (2013) which adopts the model in pricing CO_2 options.

We show how to construct the distributions of option prices directly from the posterior distributions of parameters for the two models investigated. Another key contribution of the study is that we show how to measure parameter estimation risk, and how significant it is in empirical practices. We apply a VaR-type measure for parameter estimation risk exposure in option pricing, and we show that model risk is asymmetric to buyers and sellers of options. By applying Bayesian MCMC techniques to the Greek parameters, we show that the impact of parameter estimation risk can be very significant to hedging activities. Finally, we describe how to apply this Bayesian approach to the Merton's Credit Risk model with a focus on investigating the impact of parameter estimation risk in computing the probability of default.

The rest of the paper presents as follow: Section 2 provides a summary of literature review; Section 3 reviews briefly the MJD model; Section 4 introduces the Bayesian econometrics and MCMC simulation techniques; Section 5 shows the empirical application results of both the BS and MJD models under the Bayesian estimation approach and a VaR-type measure for parameter estimation risk in option pricing; Section 6 demonstrates the application of Bayesian econometrics in the Merton's Credit Risk model; and Section 7 provides summary conclusions.

2. Literature review

There has been an array of evidence that the BS model is not consistent with empirical data (Das & Sundaram, 1999; Merton, 1976; Jorion, 1988; Drost, Nijman, & Werker, 1998; Backus, Foresi, & Wu, 2004; Batten & Ellis, 2005). The model suggests a normal distribution of stock return, whereas empirical evidence, as we know it, generally shows excessive kurtosis and negative skewness.

The MJD model developed by Merton (1976) is a key extension of the BS model. Several studies suggest that the anomalies of market return could be a result of jump events, and large price jumps are observed in market return data; see Das and Sundaram (1999), Drost et al. (1998), Jarrow and Rosenfeld (1984), Kim, Oh, and Brooks (1994) and Maekawa, Lee, Morimoto, and Kawai (2008). Burger and Kliaris (2013) argue that while the diffusion process captures the volatility generated by trading activities, the jump component captures more significant changes of stock prices generated by new information. The jump component also generates skewness and kurtosis to the stock return distribution as discussed by Das and Sundaram (1999), Gardoń (2011), Bates (1996).

Estimating the parameters of the MJD model is not a straightforward exercise because under this model the stock return distribution is an infinite mixture of normal distributions. Even under the simplified Bernoulli-Jump Diffusion setting proposed by Ball and Torous (1985), in which a maximum of one jump can occur during one unit time interval, the likelihood function is still unbounded and may have many local modes. This leads to the difficulty in estimating the parameter values using the maximum likelihood method (Kostrzewski, 2014). Due to this issue, many empirical studies of the MJD model show unreasonable large number of jumps: 162 per annum (Hanson & Westman, 2002), 179 per annum (Ramezani & Zeng, 1998), 142 per annum (Honore, 1998). Honore (1998) suggests that this issue can be circumvented by treating the jump magnitude as a constant input to the model. However, the option pricing results under such strict constraints can hardly show any improvement compared to the BS model pricing results. Frühwirth-Schnatter (2006) and Kostrzewski (2014) show that MCMC Bayesian econometrics framework can provide a better solution to this calibration problem.

Parameter estimation risk is never a trivial problem in financial modelling. Focused on asset pricing and risk management, Chung, Hui, and Li (2013) account for parameter estimation risk in equity pricing models by calculating the Bayesian posterior standard deviation of parameters, and they conclude that parameter uncertainty is sufficient to explain the price discrepancy between Chinese A- and H-share prices. Jacquier, Polson, and Rossi (1994), Bunnin et al. (2002) and Gupta and Reisinger (2014) also emphasise the importance of parameter estimation risk in option pricing and suggest the Bayesian estimation approach through MCMC computational techniques as a solution. Butler and Schachter (1997), Christoffersen and Gonçalves (2005), and Kerkhof et al. (2010) report value-at-risk