Section sigma models coupled to symplectic duality bundles on Lorentzian four-manifolds

C. I. Lazaroiu\textsuperscript{1} and C. S. Shahbazi\textsuperscript{2}

\textsuperscript{1} Center for Geometry and Physics, Institute for Basic Science, Pohang 790-784, Republic of Korea, E-mail: calin@ibs.re.kr
\textsuperscript{2} Department of Mathematics, University of Hamburg, Germany, E-mail: carlos.shahbazi@uni-hamburg.de

Abstract: We give the global mathematical formulation of a class of generalized four-dimensional theories of gravity coupled to scalar matter and to Abelian gauge fields. In such theories, the scalar fields are described by a section of a surjective pseudo-Riemannian submersion $\pi$ over space-time, whose total space carries a Lorentzian metric making the fibers into totally-geodesic connected Riemannian submanifolds. In particular, $\pi$ is a fiber bundle endowed with a complete Ehresmann connection whose transport acts through isometries between the fibers. In turn, the Abelian gauge fields are “twisted” by a flat symplectic vector bundle defined over the total space of $\pi$. This vector bundle is endowed with a vertical taming which locally encodes the gauge couplings and theta angles of the theory and gives rise to the notion of twisted self-duality, of crucial importance to construct the theory. When the Ehresmann connection of $\pi$ is integrable, we show that our theories are locally equivalent to ordinary Einstein-Scalar-Maxwell theories and hence provide a global non-trivial extension of the universal bosonic sector of four-dimensional supergravity. In this case, we show using a special trivializing atlas of $\pi$ that global solutions of such models can be interpreted as classical “locally-geometric” U-folds. In the non-integrable case, our theories differ locally from ordinary Einstein-Scalar-Maxwell theories and may provide a geometric description of classical U-folds which are “locally non-geometric”.

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1. Introduction

The construction of four-dimensional supergravity theories usually found in the physics literature (see, for example, \cite{1,2,3,4}) is local in the sense that it is carried out ignoring the topology of the space-time manifold and without specifying the precise global description of the configuration space or the global mathematical structures required to define it. Such constructions are discussed traditionally only on a contractible subset $U$ of space-time, which guarantees that any fiber bundle defined on $U$ is trivial and