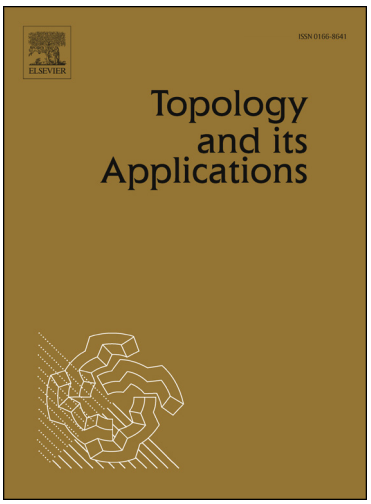
Accepted Manuscript



Connected neighborhoods in products

Alejandro Illanes, Jorge M. Martínez-Montejano, Karen Villarreal

PII: S0166-8641(18)30207-4
DOI: https://doi.org/10.1016/j.topol.2018.03.028
Reference: TOPOL 6447

To appear in: *Topology and its Applications*

Received date: 30 November 2016
Revised date: 25 July 2017
Accepted date: 5 August 2017

Please cite this article in press as: A. Illanes et al., Connected neighborhoods in products, *Topol. Appl.* (2018), https://doi.org/10.1016/j.topol.2018.03.028

CONNECTED NEIGHBORHOODS IN PRODUCTS

ALEJANDRO ILLANES*, JORGE M. MARTÍNEZ-MONTEJANO,
AND KAREN VILLARREAL

Abstract. Let X and Y be metric continua. We consider the following property (*): if M is a subcontinuum of $X \times Y$ such that $\pi_X(M) = X$ and $\pi_Y(M) = Y$, where π_X and π_Y are the respective projections on X and Y, then M has small connected neighborhoods in $X \times Y$. Property (*) has been studied by D. P. Bellamy, J. M. Łysko and the first named author. In this paper we continue studying property (*) in products of continua. We prove: (a) the product of homogeneous continua having the fixed point property has property (*); (b) the product of a solenoid and any Knaster continuum has property (*); (c) there exists a Kelley continuum X such that $X \times [0,1]$ does not have property (*); and (d) the product of a chainable Kelley continuum and $[0,1]$ has property (*).

1. Introduction

A *continuum* is a compact connected metric space, a *mapping* is a continuous function. Given a family of metric continua $\{X_\alpha : \alpha \in J\}$, the product $X = \prod_{\alpha\in J} X_\alpha$ has the following property: *full projection implies arbitrary small connected open neighborhoods (fupcon)* provided that for every subcontinuum M and open subset U of X such that $M \subset U$ and $\pi_\alpha(M) = X_\alpha$ for each $\alpha \in J$ (π_α is the α^{th}-projection), there exists an open connected subset V of X such that $M \subset V \subset U$.

Clearly, each product of locally connected continua has fupcon property.

A subcontinuum M of a continuum X is *ample* provided that for each open subset U of X with $M \subset U$, there exists a subcontinuum L of X such that $M \subset \operatorname{int}_X(L) \subset L \subset U$. So X is connected im kleinen at a point $p \in X$ provided that $\{p\}$ is ample. By [6, Lemma 1], the product $X = \prod_{\alpha\in J} X_\alpha$ has fupcon property provided that each subcontinuum M of X projecting onto each X_α is an ample subset of X.

Ample subcontinua where introduced in [12] and they have been useful to improve the understanding of homogeneous continua.

It is easy to show that if M is an ample subcontinuum of a continuum X, then the hyperspace $C(X)$ of subcontinua of X (with the Hausdorff metric) is connected im kleinen at M. In fact, when X is a Kelley continuum (see Section 4), $C(X)$ is connected im kleinen at an element $M \in C(X)$ if and only if M is ample. Thus, if X is a product with fupcon property, then it is possible to find subcontinua M of X at which $C(X)$ is connected im kleinen. This is something remarkable, since in

2010 *Mathematics Subject Classification.* primary 54F15, secondary 54F50.

Key words and phrases. chainable, continuum, product, Kelley continuum, Knaster continuum, product, pseudo-arc, solenoid.

*Corresponding author.