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Neighbor sum distinguishing total coloring and list neighbor sum distinguishing total coloring

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ABSTRACT

Let $\chi_{\Sigma}^{\ell}(G)$ and $\chi_{\Sigma}^t(G)$ be the neighbor sum distinguishing total chromatic and total choice numbers of a graph G , respectively. In this paper, we present some new upper bounds of $\chi_{\Sigma}^{\ell}(G)$ for ℓ -degenerate graphs with integer $\ell \geq 1$, and of $\chi_{\Sigma}^t(G)$ for 2-degenerate graphs. As applications of these results, (i) for a general graph G , $\chi_{\Sigma}^t(G) \leq \chi_{\Sigma}^{\ell}(G) \leq \max\{\Delta(G) + \lfloor \frac{3\text{col}(G)}{2} \rfloor - 1, 3\text{col}(G) - 2\}$, where $\text{col}(G)$ is the coloring number of G ; (ii) for a 2-degenerate graph G , we determine the exact value of $\chi_{\Sigma}^t(G)$ if $\Delta(G) \geq 6$ and show that $\chi_{\Sigma}^t(G) \leq 7$ if $\Delta(G) \leq 5$.

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1. Introduction

For terminology and notations not defined here we follow [3]. Let $G = (V, E)$ be a simple graph and $\phi : V \cup E \rightarrow [k]$ be a total- $[k]$ -weighting where $[k] = \{1, \dots, k\}$. ϕ is a neighbor sum distinguishing total- $[k]$ -weighting (abbreviated as NSD total- $[k]$ -weighting) if $\phi(u) + \sum_{e \in E(u)} \phi(e) \neq \phi(v) + \sum_{e \in E(v)} \phi(e)$ for each edge $uv \in E$, where $E(x)$ is the set of edges incident with x for a vertex x . The NSD total weighting problem was introduced by Przybyło and Woźniak [18] as a variation of a similar problem for edge weighting introduced by Karoński, Łuczak and Thomason [12]. Przybyło and Woźniak [18] conjectured that every graph has an NSD total- $[2]$ -weighting while Karoński, Łuczak and Thomason [12] conjectured that every graph without isolated edges has an NSD edge- $[3]$ -weighting. Those two conjectures are known as the 1–2 Conjecture and the 1–2–3 Conjecture, respectively. Kalkowski, Karoński and Pfender [11] showed that every connected graph with at least three vertices has an NSD edge- $[5]$ -weighting. Kalkowski [10] showed that every graph has an NSD total- $[3]$ -weighting. The readers are referred to a survey paper [20] for more information.

Recently both NSD total weighting and NSD edge weighting have been extended to choosability version. In [2], Bartnicki et al. considered the choosability version of 1–2–3 Conjecture. A graph is said to be NSD edge- k -weight-choosable if for every list assignment L which assigns to each edge e a set $L(e)$ of k real numbers, G has an NSD edge weighting ϕ such that $\phi(e) \in L(e)$ for each edge e . Przybyło and Woźniak [19] and Wong and Zhu [22] independently introduced the list version of NSD total weighting. Let $L(x)$ be a list of k real numbers assigned to each element $x \in V \cup E$ of a graph G , and let L be a collection of all these lists. Such L is called a total- k -list assignment of G . We say that G has an NSD total- L -weighting if there is an NSD total weighting $\phi : V \cup E \rightarrow \cup_{L(x) \in L} L(x)$ such that $\phi(x) \in L(x)$ for each $x \in V \cup E$. A graph G is NSD total- k -weight-choosable if for each total- k -list assignment L , G has an NSD total- L -weighting. Przybyło and Woźniak [19] conjectured that the 1–2 Conjecture is also true for the list version. That is every graph is NSD total-2-weight-choosable. This conjecture was verified

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for trees, cycles, and complete graphs by Przybyłó [16] and Wong and Zhu [22] independently. In [23], Wong and Zhu proved that every graph is NSD total-3-weight-choosable.

The (list) edge coloring version of the problem where the edge weighting is a (list) edge coloring has been studied for recent years (see, for example, [7–9,17,21]). In this paper, we study the (list) total coloring version of the problem. Let $k \geq 1$ be an integer and L be a total- k -list assignment of a graph G . A total coloring ϕ is called a total- L -coloring if $\phi(x) \in L(x)$ for each $x \in V \cup E$. Such ϕ is neighbor sum distinguishing (NSD) if $\phi(u) + \sum_{e \in E(u)} \phi(e) \neq \phi(v) + \sum_{e \in E(v)} \phi(e)$ for each $uv \in E$. The NSD total choice number, denoted by $\chi_{\Sigma}^L(G)$, is the smallest integer k such that for each total- k -list assignment L , G has an NSD total- L -coloring. In particular, a total- L -coloring is said to be a total- $[k]$ -coloring if $L(x) = [k]$ for each $x \in V \cup E$. The NSD total chromatic number of G , denoted by $\chi_{\Sigma}^t(G)$, is the smallest integer k such that G has an NSD total- $[k]$ -coloring. The concept of NSD total- $[k]$ -coloring was introduced by Piłśniak and Woźniak [15], who proposed the following conjecture.

Conjecture 1.1 ([15]). *For a graph G with maximum degree Δ , $\chi_{\Sigma}^t(G) \leq \Delta + 3$.*

Piłśniak and Woźniak [15] showed that **Conjecture 1.1** holds for complete graphs, bipartite graphs and graphs with maximum degree 3. Li et al. [13] proved that **Conjecture 1.1** holds for planar graphs with maximum degree at least 13. Recently, Li et al. [14] proved the following result.

Theorem 1.2 ([14]). *Conjecture 1.1 holds for K_4 -minor-free graphs.*

The maximum average degree of G is $\text{mad}(G) = \max\{\frac{2|E(H)|}{|V(H)|} : H \subseteq G\}$. Dong and Wang [6] showed the following theorem.

Theorem 1.3 ([6]). *Let G be a graph with at least two vertices. If $\text{mad}(G) < 3$, then $\chi_{\Sigma}^t(G) \leq \max\{\Delta + 2, 7\}$.*

The coloring number of a graph G , denoted by $\text{col}(G)$, is the least integer k such that G has a vertex enumeration in which each vertex is preceded by fewer than k of its neighbors. By using combinatorial nullstellensatz, Ding et al. [5] presented an upper bound of $\chi_{\Sigma}^t(G)$ as follows.

Theorem 1.4 ([5]). *For a graph G with at least two vertices, $\chi_{\Sigma}^t(G) \leq 2\Delta + \text{col}(G) - 1$.*

In this paper, we improve the upper bound in **Theorem 1.4** by proving its list version.

Theorem 1.5. *For a graph G with maximum degree Δ ,*

$$\chi_{\Sigma}^t(G) \leq \chi_{\Sigma}^L(G) \leq \begin{cases} \max\{\Delta + \lfloor \frac{3\text{col}(G)}{2} \rfloor - 1, 3\text{col}(G) - 2\}, & \text{if } \text{col}(G) \leq 3; \\ \max\{\Delta + \lfloor \frac{3\text{col}(G)}{2} \rfloor - 2, 3\text{col}(G) - 2\}, & \text{if } \text{col}(G) \geq 4. \end{cases}$$

For a positive integer ℓ , G is ℓ -degenerate if every subgraph of G contains a vertex of degree at most ℓ . Note that every graph G is $(\text{col}(G) - 1)$ -degenerate. For a 2-degenerate graph G , we determine the exact value of $\chi_{\Sigma}^t(G)$ when $\Delta \geq 6$.

Theorem 1.6. *Let G be a 2-degenerate graph with maximum degree Δ . Then*

- (1) *If $\Delta \leq 5$, then $\chi_{\Sigma}^t(G) \leq 7$.*
- (2) *If $\Delta \geq 6$, then*

$$\chi_{\Sigma}^t(G) = \begin{cases} \Delta + 1, & \text{if } G \text{ contains no two adjacent } \Delta\text{-vertices;} \\ \Delta + 2, & \text{otherwise.} \end{cases}$$

Note that trees, K_4 -minor-free graphs, parallel series graphs, planar graphs with girth at least 6 and graphs with $\text{mad}(G) < 3$ are all 2-degenerate. Therefore, **Theorem 1.6** implies **Theorems 1.2** and **1.3**.

Theorem 1.5 is a direct corollary of **Theorem 1.7** on ℓ -degenerate graphs.

Theorem 1.7. *Let $\ell \geq 1$ be an integer and G be an ℓ -degenerate graph with maximum degree Δ . Then*

$$\chi_{\Sigma}^L(G) \leq \begin{cases} \max\{\Delta + \lfloor \frac{3\ell + 1}{2} \rfloor, 3\ell + 1\}, & \text{if } \ell \leq 2; \\ \max\{\Delta + \lfloor \frac{3\ell - 1}{2} \rfloor, 3\ell + 1\}, & \text{if } \ell \geq 3. \end{cases}$$

We believe that the list version of **Conjecture 1.1** is also true.

Conjecture 1.8. *For a graph G with maximum degree Δ , $\chi_{\Sigma}^L(G) \leq \Delta + 3$.*

Theorem 1.7 implies that **Conjecture 1.8** is true for 2-degenerate graphs with $\Delta \geq 4$. Note that if $\Delta = 1$ then $\chi_{\Sigma}^L(G) = \chi_{\Sigma}^t(G) = 3$, and if G contains two adjacent Δ -vertices then $\chi_{\Sigma}^L(G) \geq \chi_{\Sigma}^t(G) \geq \Delta + 2$. Since all trees are 1-degenerate, by **Theorem 1.7**, we have the following corollary.

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