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# Financial management in inventory problems: Risk averse vs risk neutral policies

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## ABSTRACT

In this work, we discuss the effect of risk measure selection in the determination of inventory policies. We consider an inventory system characterized by the loss function of Luciano et al. [2003. VaR as a risk measure for multi-period static inventory models. *International Journal of Production Economics* 81–82, 375–384]. We derive the optimization problems faced by risk neutral, quadratic utility, mean-absolute and CVaR decision makers. Results show that while the global nature of the optimal policy is assured for risk coherent and risk neutral decision makers, the convexity of the quadratic utility problem depends on the stochastic properties of demand. We investigate the economic and stochastic determinants of the different policies. This allows us to establish the conditions under which each type of decision maker is indifferent to imprecision in the distribution families. Finally, we discuss the numerical impact of the choice of the risk measure by means of a multi-item inventory. The introduction of an approach based on Savage Scores allows us to offer a quantitative measurement of the similarity/discrepancy of policies reflecting different risk attitudes.

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## 1. Introduction

The purpose of this work is to investigate the quantitative implications of the risk measure choice on optimal inventory policies. We introduce a structured approach to allow the determination of the extent of the discrepancies in the policies selected by decision makers with different risk attitudes—in particular, we compare risk neutral policies to the policies of decision makers selecting variance, absolute deviation (MAD) and conditional value at risk (CVaR) as risk measures.<sup>1</sup>

Relevant literature in the last 20 years has evidenced the importance of financial and decision theoretical aspects in inventory management. The works of Grubbström and Thorstenson (1986), Thorstenson (1988), Luciano (1998), Luciano and Peccati (1999), Luciano et al. (2003) and Koltai (2006) focus on applications of the discounted cash flow methodology to inventory policies. Bogataj and Hvalica (2003) propose to utilize, besides an expected value criterion, the maximin approach. The maximin approach to the newsvendor problem is discussed in Gallego and Moon (1993) and Gallego et al. (2001). Earlier, alternative optimization criteria for the newsvendor problem have been studied in Lau (1980). This lines of thought can be seen as leading to the recent formulation of inventory management problems in terms of coherent risk measures (Chen et al., 2005; Ahmed et al., 2007; Gotoh and Takano, 2007). Chen et al. (2005) analyze risk aversion in inventory problems comparing risk measures and expected utility optimization. Ahmed et al. (2007) derive the structure of the solution of coherent risk measure optimization for the newsvendor loss function.

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<sup>1</sup> We refer to Ogryczak and Ruszczyński (1999) for the relationship between stochastic dominance and mean-risk models, to Artzner et al. (1999) for a definition of coherent risk measures and to Rockafellar and Uryasev (2002) for the definition of CVaR. A definition of mean-absolute deviation can be found in Ruszczyński and Shapiro (2005), who also discuss in detail the theory of risk measure optimization.

Gotoh and Takano (2007) discuss the solution of the newsvendor problem with CVaR. Common feature in these works is the utilization of the newsvendor loss function. Distinctive features are, in Gotoh and Takano (2007) CVaR minimization with the extension of the loss function to a multi-item single-period problem, and, in Ahmed et al. (2007), the treatment of the single-item multi-period (finite horizon) problem.

In this work, we consider a multi-item inventory system whose financial characteristics are described by the profit function of Luciano et al. (2003)—“LCP model” from now on. In order to assess the effect of alternative risk aversion representations, we are faced with formulating and studying the optimization problems of the four decision makers in the presence of the LCP model. Results show that while a risk neutral and a coherent risk averse optimal policy is always a global one, the conditions under which a mean-variance decision maker’s optimal policy is globally optimal are determined by the stochastic properties of demand.

We then investigate the determinants of the optimal policies. By deriving the analytical expressions of the optimization problems, we identify and discuss the stochastic properties that are needed by the four types of inventory managers to identify the optimal policies. This allows us to determine the conditions under which the decision makers are insensitive to imprecision in the demand distributions. As far as economic aspects are concerned, findings show that while a risk neutral policy can be determined based on the sole knowledge of revenues and variable costs, risk averse decision makers need the additional knowledge of the system fixed costs.

We then carry out a numerical discussion aimed at highlighting the quantitative differences among the optimal policies selected by the different decision makers. To compare the policy structures we introduce a methodology based on Savage’s score correlation coefficients (Iman and Conover, 1987). The numerical impact of imprecision in the demand distribution is assessed by confronting numerical results obtained with finite support distributions (Beta) to the results obtained via an infinite support distribution (Gamma).

The remainder of the paper is organized as follows. Section 2 illustrates the problem settings in the presence of a generic loss function. Section 3 presents the problem settings for the LPC profit function. In particular, Section 3.1 discusses the optimization problem for a risk neutral decision maker. Section 3.2 derives the optimization problem for a quadratic utility risk averse decision maker. Section 3.3 discusses the problem for an inventory manager utilizing MAD. Section 3.4 presents the optimization problem for a CVaR decision maker. Section 4 compares the different problems, discusses numerical results and evaluates the effect of imprecision in the demand distributions. Section 5 offers conclusions.

**2. Problems settings for generic inventories**

In this Section, we present a brief overview of risk measure optimization, introducing results that are relevant in the remainder of the paper.

We start with considering a real valued random variable,  $Z = f(\mathbf{x}, \omega)$  that depends on decision vector  $(\mathbf{x} \in \mathbb{R}^N)$  and  $\omega \in \Omega$ , where  $(\Omega, \mathcal{B}(\Omega), P)$  is a measure space (Ruszczynski and Shapiro, 2005). If  $Z$  represents a loss or a disutility (Ruszczynski and Shapiro, 2005) the optimal risk neutral choice solves the stochastic programming problem:

$$\min_{\mathbf{x} \in S} \mathbb{E}_P[f(\mathbf{x}, \omega)] \tag{1}$$

where  $S$  is the feasible set.

Many authors, have questioned expected value optimization, as the resulting policy is optimal “on average (Ruszczynski and Shapiro, 2005).” Indeed, the most general formulation of an optimization problem is in terms of expected utility maximization. The problem is stated as

$$\max_{\mathbf{x} \in S} \mathbb{E}_P[u(f(\mathbf{x}, \omega))] \tag{2}$$

The utility function,  $u(\cdot)$ , captures the decision maker preferences, giving full consideration to His/Her risk aversion/proneness. However, the form of  $u(\cdot)$  can be “extremely difficult to elicit (Ruszczynski and Shapiro, 2005).” A first way to go around such a difficulty is to pre-determine the shape of the utility function (see Chen et al., 2005), or to approximate it through a series expansion. When the expansion is arrested at the second order one obtains the quadratic assumption which is at the basis of Markowitz’s (1952) portfolio selection model. A decision maker possessing a quadratic utility function, ought to select  $\mathbf{x}$  such that

$$\min_{\mathbf{x} \in S} \mathbb{V}_P[f(\mathbf{x}, \omega)] \tag{3}$$

As second way to take risk aversion into consideration, which has been successfully proposed in the finance literature in the seminal work of Artzner et al. (1999) consists of making use of coherent measures of risk. Let  $\rho(Z)$  denote a function satisfying the following axioms of Artzner et al. (1999):

- (1) Translational invariance:  $\rho[Z + a] = \rho[Z] + a$ .
- (2) Subadditivity:  $\rho[Z_1 + Z_2] \leq \rho[Z_1] + \rho[Z_2]$ .
- (3) Positive homogeneity:  $\forall \alpha > 0 \rho[\alpha X] = \alpha \rho[X]$ .
- (4) Monotonicity: Given  $Z_1, Z_2$  such that  $Z_1(\omega) \geq Z_2(\omega) \forall \omega \in \Omega$  then  $\rho[Z_2] \leq \rho[Z_1]$ .
- (5)  $\forall Z \neq 0, \rho[Z] > 0$ .

Then,  $\rho(Z)$  is a coherent measure of risk and a decision maker modeling risk aversion through  $\rho(Z)$  solves the following problem (Artzner et al., 1999; Ruszczynski and Shapiro, 2005):

$$\min_{\mathbf{x} \in S} \rho[f(\mathbf{x}, \omega)] \tag{4}$$

For a complete commentary on the meaning of the axioms, we refer to Artzner et al. (1999). We, however, place emphasis on the following results that provide the background for the findings presented in the next sections.

**Remark 1.** 1. The axioms of subadditivity and positive homogeneity lead to the convexity of  $\rho[f(\mathbf{x}, \omega)]$ .

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