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Quantifying fluctuations in economic systems by adapting methods of statistical physics

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Abstract

The emerging subfield of econophysics explores the degree to which certain concepts and methods from statistical physics can be appropriately modified and adapted to provide new insights into questions that have been the focus of interest in the economics community. Here we give a brief overview of two examples of research topics that are receiving recent attention. A first topic is the characterization of the dynamics of stock price fluctuations. For example, we investigate the relation between trading activity – measured by the number of transactions $N_{\Delta t}$ – and the price change $G_{\Delta t}$ for a given stock, over a time interval $[t, t+\Delta t]$. We relate the time-dependent standard deviation of price fluctuations – volatility – to two microscopic quantities: the number of transactions $N_{\Delta t}$ in Δt and the variance $W^2_{\Delta t}$ of the price changes for all transactions in Δt. Our work indicates that while the pronounced tails in the distribution of price fluctuations arise from $W_{\Delta t}$, the long-range correlations found in $|G_{\Delta t}|$ are largely due to $N_{\Delta t}$. We also investigate the relation between price fluctuations and the number of shares $Q_{\Delta t}$ traded in Δt. We find that the distribution of $Q_{\Delta t}$ is consistent with a stable Lévy distribution, suggesting a Lévy scaling relationship between $Q_{\Delta t}$ and $N_{\Delta t}$, which would provide one explanation for volume-volatility co-movement. A second topic concerns cross-correlations between the price fluctuations of different stocks. We adapt a conceptual framework, random matrix theory (RMT), first used in physics to interpret statistical properties of nuclear energy spectra. RMT makes predictions for the statistical properties of matrices that are universal, that is, do not depend on the interactions between the elements comprising the system. In physics systems, deviations from the predictions of RMT provide clues regarding the mechanisms controlling the dynamics of a given system, so this framework can be of potential value if applied to economic systems. We discuss a systematic comparison between the statistics of the cross-correlation matrix **C** – whose elements C_{ij} are the correlation-coefficients between the returns of stock i and j – and that of a random matrix having the same symmetry properties. Our work suggests that RMT can be used to distinguish random and non-random parts of **C**; the non-random part of **C**, which deviates from RMT results provides information regarding genuine cross-correlations between stocks.

Keywords: Volatility; Econophysics; Random matrix theory; Cross-correlations

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1. Introduction

The interest of physicists in economic systems has roots that date back at least as far as 1936, when the Italian physicist Majorana wrote a paper on the analogy between statistical laws in physics and in the social sciences [1]. Majorana's intriguing point of view was initially considered of marginal interest since, until recently, not many professional physicists performed research associated with social or economic systems (for exceptions see, e.g., Refs. [2–13]).

Physics research activity in this field has become less episodic and a research community is beginning to emerge (see Refs. [14–18] for details). The hope of physicists is that their efforts could in time provide an approach complementary to the approaches in economics, particularly since a number of economists are working along parallel lines [5,13,15,19–23].

Our group's recent econophysics research focuses broadly on two lines of study. The first focus relates to the statistical characterization of the "microscopic" dynamics of stock returns [2,24–54]. The second focus relates to the study of cross-correlations between the returns of stocks [55–72].

2. Scaling and universality: two concepts of modern statistical physics

Statistical physics deals with systems comprising a very large number of interacting subunits, for which predicting the exact behavior of the individual subunit would be impossible. Hence, one is limited to making statistical predictions regarding the collective behavior of the subunits. Recently, it has come to be appreciated that many such systems which consist of a large number of interacting subunits obey universal laws that are independent of the microscopic details. The finding, in physical systems, of universal properties that do not depend on the specific form of the interactions gives rise to the intriguing hypothesis that universal laws or results may also be present in economic and social systems [2,14]. [1]

2.1. Background

Suppose we have a small bar magnet made up of, say, 10^{12} strongly interacting subunits called "spins". We know it is a magnet because it is capable of picking up thumbtacks, the number of which is called the order parameter M. As we heat this system, M decreases and eventually, at a certain critical temperature T_c, it reaches zero. In fact, the transition is remarkably sharp, since M approaches zero at T_c with

[1] An often-expressed concern regarding the application of physics methods to the social sciences is that physical laws are said to apply to systems with a very large number of subunits (of order of $\approx 10^{20}$) while social systems comprise a much smaller number of elements. However, the "thermodynamic limit" is reached in practice for rather small systems. For example, in early computer simulations of gases or liquids reasonable results are already obtained for systems with 20–30 atoms.