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Optimal replenishment order for uncertain demand in three layer supply chain

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ABSTRACT

The authors study the well-known order quantity model in a three-player context, using a framework of newsboy problem. An expected average cost function of the chain by trading off inventory cost and shortage costs is formulated which is minimized to obtain optimal order sizes of the supplier, manufacturer and retailer. Our theoretical analysis of both cases; (i) when demand per unit time of each member of the chain is uncertain, (ii) when uncertain demand is distributed uniformly over finite time horizon; suggests the determining of optimal order sizes of the members so that the expected average channel cost is minimum. Numerical examples for different distributions are illustrated to justify our model.

1. Introduction

It is common belief to all enterprises in a multi-echelon supply chain that the channel cost in a centralized supply chain is more than the channel cost in a decentralized system. In the centralized supply chain system, one member of the chain is a decision maker and the others are his followers whereas, in the decentralized system, all parties of the chain are decision makers of the joint businesses. The two main critical factors among the other factors in inventory management in a multi-firm supply chain context are incentive conflicts and information asymmetries. These two factors in a supply chain process often cause supplier/ manufacturer and buyer/retailer to make incorrect production and replenishment plans that result in either under or over stocking. This situation is worse for products with short-life cycles but also for the members who have invested capital to run their businesses smoothly. Many researchers have discussed the inventory model in the multi-echelon supply chain system. Among them, Lu (1995) showed a heuristic solution of the model for single-vendor and multiple buyers. Goyal (1995) developed an inventory model where successive shipment sizes from manufacturer to customer increased by a ratio of production rate and demand rate. Goyal and Gunasekaran (1995) observed an integrated production–inventory–marketing model to determine economic production quantity and economic order quantity for raw materials in a multi-echelon production system. Banerjee and Kim (1995) investigated a collaborating inventory of the buyer, manufacturer and raw material suppliers in just-in-time environment. Thomas and Grifin (1996) remarked that an efficient supply chain requires planning and coordination among the various channels. Xiao et al. (2010) proposed a game theoretic model for a three-stage supply chain consisting of one retailer, one manufacturer and one subcontractor to determine optimal order quantity, wholesale pricing and lead-time decisions, while the manufacturer produced a seasonal/perishable product. Sana (2011a) proposed a three layer supply chain involving a supplier of raw materials, manufacturer and retailer for deterministic demand. Recently, Taleizadeh et al. (2012) investigated an inventory model for a multi-product, multi-chance constraint, multi-buyer and single-vendor, considering uniform distribution demand and lot size dependent lead-time partial backlogging. Sarkar (2012a) extended an EOQ model for time-varying demand and deteriorating with discounts on purchasing costs under the environment of delay-in-payments. Sarkar (2012b) analyzed a stock-dependent inventory model considering different delay periods with finite replenishment rate. Sarkar (2012c) investigated an economic manufacturing quantity model with price and advertising sensitive demand patterns in an imperfect production process considering the effect of inflation. Other notable works in this field are those of Aderohunmu et al. (1995), Woo et al. (2001), Viswanathan and Wang (2003), Khouja (2003), Goyal et al. (2003), Cardenas-Barron (2007), Yao et al. (2007), Huang and Ye (2010), Chen and Bell (2011), among others.

Newsboy problem is simple but it is a famous model in inventory theory because of its elegance in application in business sectors. According to its formulation, a decision maker requires how many units of the goods to be stocked in order to minimize the expected cost when demand in market is uncertain. In this problem, optimal replenishment size is obtained by balancing between the expected cost of under

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stocking and the expected cost of overstocking. Khouja (1995) formulated and solved a newsboy problem with multiple discounts. Petruzzi and Dada (1999) extended the newsboy problem in which stocking units and selling prices are obtained simultaneously. A newsboy problem with a simple reservation arrangement was investigated by Chen and Chen (2009). In their model, a special discount rate is offered to the consumers who make a reservation for the product. Chen and Chen (2010) developed a model for the multiple-item budget-constraint newsboy problem incorporating a reservation policy to meet marketing needs in which discount rate is provided to those customers who are willing to make a reservation. Zhang (2010) studied the classical newsboy model by incorporating both the budget constraint and supplier quantity discounts. Lee and Hsu (2011) developed a model for the decision-maker in a distribution-free newsboy problem to determine the expenditure on advertising and the order quantity which would maximize the expected profit against the worst possible distribution of the demand. Sana (2011b) developed a stochastic economic order quantity model over finite time horizon while random demand is dependent on random sales price. Based on partial backlogging and lost sale cases during stock out situations, the author obtained an optimal replenishment size so that the integrated expected profit is maximized. Sarkar and Moon (2011) investigated a classical EPQ model with stochastic demand under the effect of inflation. A large number of noteworthy works by Johansen and Thorstenson (1993), Chen and Chuang (2000), Sana and Chaudhuri (2002), Sana and Chaudhuri (2005), Abdel-Malek and Montanari (2005), Matsuyama (2006), Chou and Chung (2009), Wang (2010) and Hsieh and Lu (2010) should be mentioned, among others.

In this paper, we consider a three layer supply chain model involving a supplier, manufacturer and retailer as members of the chain. Here, supply rate of raw materials, production rate of manufacturer, and the rate of demand of retailer and customers are random in character which follow various types of distribution functions. The supplier supplies raw materials to the manufacturer who produces finished products which are delivered to the retailer, and retailer sales the items to the customers. At each stage of the channel, stock out situation may occur. Finally, an expected average cost function of the collaborating system by trading off inventory costs and shortage costs is formulated. At first, we develop the model for the demand per unit time at each stage. Then, we develop the model for the demand when it is distributed uniformly over a finite horizon.

2. Fundamental assumptions and notations

The following assumptions and notations are assumed to develop the model.

2.1. Assumptions

1. The model is developed for a single item.
2. The lead time at each stage is neglected.
3. The demand rate is assumed to be a random variable.
4. Shortages may occur due to uncertain demands.

2.2. Notation

Q_s	ordering size of the supplier,
Q_m	production lot size of the manufacturer,
Q_r	ordering size of the retailer,
α	probability of $Q_s \le Q_m$,
β	probability of $Q_m \le Q_r$,
C_h^s	holding cost per unit per unit time at the supplier,
C_s^s	shortage cost per unit per unit time at the supplier,
C_h^m	holding cost per unit per unit time at the manufacturer,
C_s^m	shortage cost per unit per unit time at the manufacturer,
C_h^r	holding cost per unit per unit time at the retailer,
C_s^r	shortage cost per unit per unit time at the retailer,
$f_m(x)$	density function of random supply rate (x) to the manufacturer,
$f_r(y)$	density function of random demand rate (y) of the retailer,
$f_c(z)$	density function of random demand rate (z) of the customers at the retailer,
$EACS$	individual expected average cost of the supplier,
$EACM$	individual expected average cost of the manufacturer,
$EACR$	individual expected average cost of the retailer,
$EACI$	integrated expected average cost of the chain.

3. Formulation of the model

3.1. Model-I

In this model, the supplier receives order size Q_s from outside suppliers, the manufacturer produces lot size Q_m and the retailer receives order size Q_r from the manufacturer. Here, demand rate x of the manufacturer is a random variable that follows the density function $f_m(x)$. The demand rate y of the retailer is a random variable that follows the density function $f_m(y)$. The demand rate z of the customers is a random variable that follows the density function $f_c(z)$. Let, *Prob.* $(Q_m > Q_s) = \alpha$ and *Prob.* $(Q_r > Q_m) = \beta$, then, *Prob.* $(Q_m \le Q_s) = 1-\alpha$ and *Prob.* $(Q_r \le Q_m) = 1-\beta$. Now, the individual expected average costs of the supplier, manufacturer and retailer are as follows:

$$\begin{aligned} EACS &= \alpha\left[C_h^s \int_0^{Q_s}(Q_s - x)f_m(x)dx + C_s^s \int_{Q_s}^{Q_m}(x - Q_s)f_m(x)dx\right] \\ &\quad + (1-\alpha)C_h^s \int_0^{Q_s}(Q_s - x)f_m(x)dx = C_h^s \int_0^{Q_s}(Q_s - x)f_m(x)dx \\ &\quad + \alpha C_s^s Max\left[\int_{Q_s}^{Q_m}(x - Q_s)f_m(x)dx, 0\right], \end{aligned} \tag{1}$$

$$\begin{aligned} EACM &= \beta\left[C_h^m \int_0^{Q_m}(Q_m - y)f_r(y)dy + C_s^m \int_{Q_m}^{Q_r}(y - Q_m)f_r(y)dy\right] \\ &\quad + (1-\beta)C_h^m \int_0^{Q_m}(Q_m - y)f_r(y)dy = C_h^m \int_0^{Q_m}(Q_m - y)f_r(y)dy \\ &\quad + \beta C_s^m Max\left[\int_{Q_m}^{Q_r}(y - Q_m)f_r(y)dy, 0\right] \end{aligned} \tag{2}$$

and

$$EACR = C_h^r \int_0^{Q_r}(Q_r - z)f_c(z)dz + C_s^r \int_{Q_r}^{\infty}(z - Q_r)f_c(z)dz. \tag{3}$$

Therefore, the expected average cost of the channel is $EACI = EACS + EACM + EACR$. Now, simplifying this function by using the probabilities, we have

$$EACI = \begin{Bmatrix} EA_1, \; Q_r \ge Q_m \ge Q_s \\ EA_2, \; Q_r \le Q_m \ge Q_s \\ EA_3, \; Q_r \ge Q_m \le Q_s \\ EA_4, \; Q_r \le Q_m \le Q_s \end{Bmatrix} \tag{4}$$

where

$$\begin{aligned} EA_1 &= C_h^s \int_0^{Q_s}(Q_s - x)f_m(x)dx + \alpha C_s^s \int_{Q_s}^{Q_m}(x - Q_s)f_m(x)dx \\ &\quad + C_h^m \int_0^{Q_m}(Q_m - y)f_r(y)dy + \beta C_s^m \int_{Q_m}^{Q_r}(y - Q_m)f_r(y)dy \\ &\quad + C_h^r \int_0^{Q_r}(Q_r - z)f_c(z)dz + C_s^r \int_{Q_r}^{\infty}(z - Q_r)f_c(z)dz, \end{aligned}$$