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Loss allocation of radial distribution system using Shapley value: A sequential approach

Satish Sharma *, A.R. Abhyankar

Department of Electrical Engineering, Indian Institute of Technology Delhi, 110016, India

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ABSTRACT

This paper presents a new method to allocate the power loss in a radial distribution system to loads and distributed generations (DGs). Integration of DGs has increased the importance of loss allocation because it alters the total power losses. The proposed method calculates the loss allocation in sequential manner using Shapley value method specifically for radial distribution system. The losses allocated by using proposed method are same as obtained from the conventional Shapley value method which satisfy the axioms of fairness. An important virtue of proposed method is that it reduces the memory and computational burden as well as ensures the fair allocation and recovery of total complex losses. The results are discussed and illustrated on different test systems.

1. Introduction

In deregulated power sector environment, the transmission and distribution as network owners play no part in the competition between generators and loads except providing the network access for power transfer. The specific costs associated with power delivery such as losses and cost of wheeling the energy through the network should be allocated to the generators and loads. The allocation method should be fair and based on the basis of individual network usages. The difficulty arises due to nonlinearity between losses and power flowing through network.

The distribution networks (DN) are inherently different from the transmission networks due to the passive nature of network and unidirectional power flows. They are less complicated in nature due to radial topology with large number of loads of all varieties. In recent years, a large number of DGs have been integrated into the DN. Because of this, the power flow in DN has changed into bidirectional and it becomes active in nature. The total distribution losses can be reduced by optimal placement of DGs. Hence, the loss allocation (LA) method should take care of location and role of individual participants in the distribution losses.

An extensive literature is available for LA [1–4] and cost allocation [5–7] on transmission networks. Few methods developed for the transmission network LA are also implemented with some modifications and assumptions in distribution networks. However, the slack bus plays the main distinction factor between LA methods for transmission and distribution networks [8]. The transmission losses are allocated to both, loads and generators, and hence, the transmission losses are also allocated to the slack bus. However, the slack bus in distribution system is the connection to a higher voltage network (usually to the transmission system) and it does not have a physical generator. It works as the main power source in distribution system with or without DG present. Hence, the distribution losses allocated to the slack bus will be very large and redistribution of these losses to other nodes may lead to unfair results.

According to [9], the losses in the network are due to energy transaction between generators and loads, hence effectively both generators and loads should pay for the losses in same ratio by pro-rata method. This method is easy to understand and implement, but it does not consider the network usages by individual participant. It also fails to allocate losses fairly in DN where root node supplies most of the power with DGs. Two loss allocation methods have been proposed in [10] which compute the marginal and direct loss coefficients by use of the Newton-Raphson power flow and require Jacobian and Hessian matrices. The computation of these matrices becomes very intensive in case of DN with large number of nodes or low X/R ratio. A succinct method was proposed in [11] for LA which considers both active and reactive power injected into the network. An ambiguity was found by Carpanto et al. in this method that under specific circumstances associated

* Corresponding author.
E-mail addresses: satishsharma1500@gmail.com (S. Sharma), abhyankar@ee.iitd.ac.in (A.R. Abhyankar).

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with reactive power loads, the LA will not be reliable [12]. Z-bus based LA methods have been presented in [2,13] to allocate losses to loads and generators in transmission system. For these methods, the Z-bus matrix can only be defined if the Y-bus matrix is non-singular. The shunt admittance can be negligible for the distribution networks with overhead lines only, which causes the Y-bus to be singular. In addition, the singularity of Y-bus may be very sensitive to the variation of the system parameters where the shunt capacitances are relatively low [8].

A detailed overview and classification of LA techniques for DN can be found in [12,14–17]. A three step procedure for LA has been proposed using downstream and upstream tracing which allocates losses separately to loads and then to DGs in respective two steps [14]. In third step, the remaining losses are allocated based on the updated voltage profile due to addition of DGs. Different input data has been used (power flow data for with and without DGs) in first two steps which leads to approximated results. The other tracing based approaches are used to decompose branch current [12], power [16], and energy [18] into the respective nodal injections to allocate the losses to loads and DGs.

The LA method proposed in [17] separately allocates the losses to loads and DGs using power flow results. It assigns zero loss to loads at those nodes where active generation exceeds their active demand and vice versa for losses assigned to DGs. Then, it calculates the loss allocated to other nodes for active and reactive flows. The results of this method need normalization to compensate the over recovery of the losses. The loss due to both active and reactive flow is implicitly neglected in the loss expression due to active flows to reduce the complexity.

An effective LA method should be easy to understand and implement. It should satisfy the axioms of fairness and consider the individual's contribution to the system losses. Hence, this problem can be solved using Shapley value (SV) concept [19]. The LA solution will consider the size and location of loads and DGs as Shapley value considers all the combinations and permutations of the participants. But, the drawback of Shapley value solution is that it is computationally troublesome for realistic systems.

There are few methods developed for transmission loss allocation [20–22] using cooperative game approach. In [20], a coalitional game is divided into two steps for transmission LA. In first step, branch currents are allocated to each player and then the losses in each branch are allocated using SV. The proposed method in [21] is based on weakly conditioned imputation. It calculates the response of each player in each branch which is subjective to the method used. In this case, it is obtained using superposition of currents of all players but it should be noted that the superposition of losses would not hold good. Aumann-Shapley value method was used to allocate the complex losses [22] of transmission network. The allocation procedure is based upon decomposing line currents into currents of each player, which in itself can be interpreted in multiple ways.

In this paper, we propose a sequential Shapley value (SSV) approach that does the LA for radial DN by solving Shapley value problem in sequential manner. The radial structure of DN is exploited to use SV concept with very less computational effort. SSV is a branch oriented approach and it allocates losses in forward sequence from the root node to end nodes over radial feeders. The active power losses in upward branch will be allocated to current injections from loads and downwards branches at receiving node of the branch using conventional SV in sequence. Then, in the sequence of the flow of the branch currents, the losses of successive branches are allocated.

The number of coalitions in conventional SV for any n player game is $2^n - 1$ which is very large for realistic systems. However, the number of coalitions required for solving the SSV is very small in number and depends on the network. The component of active and reactive current in active power losses can be identified from the allocated losses. The allocation using SSV is simple and easy to understand without any assumptions and approximations.

The rest of the paper is organized as follows: Section 2 explains the LA in game theoretic aspect and provides the algorithm using SV. Section 3 presents the proposed sequential approach of SV to loss allocation of radial distribution systems. The negative allocations to DGs are discussed and derived in Section 4. The generalized formulation is provided in Section 5. The results on different test systems are presented and discussed in Section 6. The contribution of proposed approach is concluded in Section 7.

2. Loss allocation through cooperative game

Game theory is the modeling of conflict and cooperation to obtain certain objective between rational decision makers. It has several practical applications in economics, social, political, logic, biology and many others. A game will be called cooperative game if it is bound by some defined contracts and collectively seeks the total wealth or certain objective towards maximization by social decision making. In the electrical power sector, game theory has been applied to many problems such as loss and cost allocation, bidding, and congestion management, etc. [22–25].

2.1. Shapley value

In a cooperative game, all players in the group play some part in maximization of total wealth of the group. Lloyd Shapley proposed the value concept in 1953 to define the importance and contribution of individual player in the group. Let any player has zero value by himself but he is more valuable in forming coalitions. Hence, players in game will form coalitions if the value of coalitions is greater than the total of individual value. The Shapley value provides the unique solution for each player. The solution satisfies the four axioms of fairness: Efficiency, Symmetry, Dummy property and Additivity.

Let the number of players in a cooperative game are n and the grand coalition will be formed as $N = \{1, 2, 3, \ldots, n\}$. Total number of coalitions are $(2^n - 1)$ excluding null or empty coalition. The characteristic function value $v(S)$ gives the wealth of a coalition S which is subset of N ($S \subset N$). The Shapley value $\phi_i(v)$ for player i can be defined as [19]:

$$\phi_i(v) = \sum_{\substack{S \subseteq N \\ i \in S}} \frac{(|S|-1)!(n-|S|)!}{n!}[v(S) - v(S - \{i\})] \tag{1}$$

where $|S|$ is the number of players in coalitions S.

2.2. Loss allocation by Shapley value

A large number of DGs have joined the distribution system recently and started to contribute the current into the grid. The placement of DGs can increase or decrease the total losses occurred in the system depending on location of their installation. Let the loads and DGs be the players in the cooperative game and losses to be allocated are due to grand coalition which consists of all players. An example of simple distribution system is shown in Fig. 1 and is used to explain the LA using conventional SV. Here, the losses due to branch current (b_1, b_2, b_3, b_4) will be distributed to loads (I_2, I_3, I_4, I_5) by SV which is calculated by following algorithm:

1. Obtain the system, loads, and DGs data.
2. Run the distribution load flow for all plausible coalitions.