



# Conditions for some balances of economic system: An input–output analysis using the spectral theory of nonnegative matrices

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## ABSTRACT

Twelve kinds of special semipositive matrices and their basic characters are presented. Employing these matrices and the previous results in Zeng (2008), we research the conditions for the balances between final output values and values-added, and between input multipliers and output multipliers in an economy. A necessary and sufficient condition that (i) there exists a unique vector of output adjustment coefficients such that (a) all sectoral final output values equal their respective sectoral values-added in the new output system, or (b) all sectoral input multipliers redefined by the new output system equal their respective sectoral output multipliers; or (ii) there exists a unique vector of price adjustment coefficients such that (a) all sectoral values-added equal their respective sectoral final output values in the new price system, or (b) all sectoral output multipliers redefined by the new price system equal their respective sectoral input multipliers; is the irreducibility of the matrix of intermediate output (or input) coefficients. A necessary and sufficient condition that (i) there is no vector of output adjustment coefficients which enables all sectoral final output values (or input multipliers) to equal their respective sectoral values-added (or output multipliers), or (ii) there is no vector of price adjustment coefficients which enables all sectoral values-added (or output multipliers) to equal their respective sectoral final output values (or input multipliers); is that the matrix of intermediate output (or input) coefficients has at least one non-final (or non-initial) class. These class relations and their equivalent conditions are summarized. The elaborate examples verify the main conclusions thoroughly.

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## 1. Introduction

Since an input–output table can accurately reflect the interdependence of industries or sectors in an economic system, the various input–output models are extensively used.

It is known that in an economic system expressed by a monetary input–output table the sum of every sectoral final output value equals the sum of every sectoral value-added, which is called the *gross national product*. In general, some sectoral final output value may be unequal to this sectoral value-added. However, theoretically it is possible that all sectoral final output values equal their respective sectoral values-added. This is one kind of *balance* of the economic system, which can be called a *balance between final output values and values-added*.

Besides, in an economic system shown via a monetary input–output table, a sectoral input (or supply) multiplier measures the rate of change of total input values throughout all sectors of the economy with respect to this sectoral value-added (cf. Miller and Blair (1985, Chapter 9)). Dually, a sectoral output (or demand) multiplier measures the rate of change of total output values throughout all sectors of the economy with respect to this sectoral final

output value. Simply speaking, the input multiplier reflects the push influence of change of primary input on the economy; the output multiplier reflects the pull influence of change of final output on the economy. Generally, some sectoral input multiplier may be unequal to this sectoral output multiplier. Nevertheless, theoretically it is possible that all sectoral input multipliers equal their respective sectoral output multipliers. Namely, the push influence of change of every sectoral primary input on the economy is exactly equivalent to the pull influence of change of the matching sectoral final output on the economy. This is another kind of *balance* of the economic system, which can be called a *balance between input multipliers and output multipliers*.

Several problems arise immediately. What is the necessary and sufficient condition for the balance between final output values and values-added? What is the necessary and sufficient condition for the balance between input multipliers and output multipliers? If the necessary conditions are satisfactory, but the necessary and sufficient conditions are not, can the necessary and sufficient conditions be satisfied via adjusting output system or price system? What is the necessary and sufficient condition that the adjustable possibility and uniqueness hold?

Zeng (2008) discussed the output adjustment model and the price adjustment model and their basic properties, and analyzed

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the effects of changes in outputs and in prices on the economic system using the spectral theory of nonnegative matrices. In this paper, based on the results in Zeng (2008), we solve the above several problems, and deduce a set of necessary and sufficient conditions for (im)possibility and (non)uniqueness of the economic adjustment that enables all sectoral final output values to equal their respective sectoral values-added, and/or all sectoral input multipliers to equal their respective sectoral output multipliers.

The paper is organized as follows.

In Section 2, some formulas on output adjustment model and price adjustment model are derived, which will be applied to the next sections. Moreover, we construct twelve kinds of special semipositive matrices whose spectral radii are all equal to one, and display their fundamental properties, which are very useful and indispensable to the next sections.

In Section 3, employing the results in Zeng (2008) and Section 2 of this paper, we pose and solve six input–output economic problems involving the balances between final output values and values-added, and between input multipliers and output multipliers, where Theorem 1 is the core. Compared with the above several problems, the six input–output economic problems are more concrete.

Section 4 expressed by Theorem 2 and its proof is a summary of the class relations for the matrix of intermediate output (or input) coefficients and the corresponding equivalent conditions.

In Section 5 the elaborate examples illustrate or verify the main conclusions thoroughly.

## 2. Preliminaries

### 2.1. Notations and terminologies

For the general notations and terminologies, let  $\neg, \wedge, \vee,$  and  $\Leftrightarrow$  stand for negation, conjunction, disjunction, and equivalence, respectively. Via  $\Gamma \Rightarrow \Omega,$  or  $\Omega \Leftarrow \Gamma,$  we denote that  $\Gamma$  implies  $\Omega.$  Let  $0$  be zero, a zero vector or zero matrix. A vector or matrix  $M > 0$  means that  $M$  is *semipositive*, i.e., every entry of  $M$  is nonnegative, and  $M \neq 0.$  A vector or matrix  $M \gg 0$  implies that  $M$  is *positive*, i.e. every element of  $M$  is positive. A vector or matrix  $M$  is called *strictly semipositive*, if  $M > 0$  and at least one entry is zero. Evidently,  $M$  is semipositive if and only if  $M$  is positive or strictly semipositive. By  $M^t$  we indicate the transpose of vector or matrix  $M.$  Let  $\rho(M)$  represent the spectral radius of a matrix  $M.$  Let  $\hat{H} = \text{diag}(H) = \text{diag}(h_1, h_2, \dots, h_n)$  be the diagonal matrix with  $h_1, h_2, \dots, h_n$  as its main diagonal entries, where  $H$  is a column or row vector. The unit matrix is represented via  $I.$  The unit column vector  $E = (1, 1, \dots, 1)^t.$  Let  $W$  denote a permutation matrix. The uniqueness of an eigenvector  $H$  of a matrix associated with an eigenvalue means that  $H$  is unique up to a scalar multiple. The uniqueness of a nonzero solution vector  $L$  of a homogeneous linear equations system means that  $L$  is unique up to a scalar multiple.

Let  $q = (q_i)_{n \times 1} \gg 0$  be a column vector of physical gross outputs;  $p = (p_j)_{1 \times n} \gg 0,$  a row vector of prices;  $X = \hat{p}q = (x_i)_{n \times 1} \gg 0,$  a column vector of gross output values (i.e. a column vector of monetary gross outputs);  $\bar{T} > 0,$  a physical transaction matrix for the intermediate products;  $T = \hat{p}\bar{T} > 0,$  a monetary transaction matrix for the intermediate products;  $\bar{F} = (\bar{f}_i)_{n \times 1} > 0,$  a column vector of physical final outputs;  $F = \hat{p}\bar{F} = (f_i)_{n \times 1} > 0,$  a column vector of final output values (i.e. a column vector of monetary final outputs);  $V = (v_j)_{1 \times n} > 0,$  a row vector of values-added;  $Y = \hat{q}^{-1}\bar{F} = \hat{X}^{-1}F = (y_i)_{n \times 1} > 0,$  a column vector of final output rates;  $R = V\hat{X}^{-1} = (r_j)_{1 \times n} > 0,$  a row vector of value-added rates;  $\bar{A} = \hat{g}^{-1}\bar{T} = \hat{X}^{-1}T > 0,$  a matrix of intermediate output coefficients;  $A = \bar{T}\hat{q}^{-1} > 0,$  a matrix of physical intermediate input coefficients;  $A = T\hat{X}^{-1} > 0,$  a matrix of monetary intermediate

input coefficients;  $\bar{B} = (I - \bar{A})^{-1} = (\bar{b}_{ij})_{n \times n} > 0,$  a Ghosh inverse matrix, where  $\bar{b}_{ij}$  can be called the *total supply coefficient of sector  $i$  to sector  $j$* ;  $\bar{B} = (I - \bar{A})^{-1} = (\bar{b}_{ij})_{n \times n} > 0,$  a physical Leontief inverse matrix;  $B = (I - A)^{-1} = (b_{ij})_{n \times n} > 0,$  a monetary Leontief inverse matrix, where  $b_{ij}$  can be called the *monetary total requirement coefficient of sector  $j$  from sector  $i$* ;  $G = \bar{B}E = (g_i)_{n \times 1},$  a column vector of input multipliers;  $D = E^tB = (d_j)_{1 \times n},$  a row vector of output multipliers;  $\alpha \geq \max_{1 \leq i \leq n} (g_i);$  where  $\rho(\bar{A}) = \rho(\bar{A}) = \rho(A) < 1.$

### 2.2. Some formulas on output adjustment model and price adjustment model

For investigating the change of output system we define the column vector of *output adjustment coefficients* to be

$$Q = \hat{q}^{-1}q^\# = (Q_1, Q_2, \dots, Q_n)^t, \tag{1}$$

where  $q^\# \gg 0$  is a column vector of the redefined new physical gross outputs. Let the superscript  $\#$  correspond to the new output system. By (1), since  $\bar{F} = (I - \bar{A})q,$   $\bar{A} = \hat{q}^{-1}\bar{A}\hat{q},$  and  $\bar{A}$  is constant via the usual hypothesis, we have  $F^\# = \hat{p}\bar{F}^\# = \hat{p}(I - \bar{A})q^\# = \hat{p}(I - \hat{q}\hat{q}^{-1})\hat{q}Q = \hat{X}(I - \bar{A})Q$  when the price system is unchanged. Namely

$$F^\# = \hat{X}(I - \bar{A})Q. \tag{2}$$

Besides,  $X^\# = \hat{p}q^\# = \hat{p}\hat{q}Q = \hat{X}Q$  and  $T^\# = \hat{p}\bar{T}^\# = \hat{p}\bar{A}\hat{q}^\# = \hat{p}\bar{A}\hat{q}Q = \hat{p}\bar{T}Q = TQ$  by (1), that is,

$$X^\# = \hat{X}Q, \tag{3}$$

$$T^\# = TQ. \tag{4}$$

Since  $V = X^t - E^tT,$  we have  $V^\# = (X^\#)^t - E^tT^\# = (X^t - E^tT)Q = VQ$  via (3) and (4), i.e.

$$V^\# = VQ. \tag{5}$$

Dually, for investigating the alteration of price system we define the row vector of *price adjustment coefficients* to be

$$P = p^*\hat{p}^{-1} = (P_1, P_2, \dots, P_n), \tag{6}$$

where  $p^* \gg 0$  is a row vector of the redefined new prices. Let the superscript  $*$  correspond to the new price system. By (6), since  $V = p(I - \bar{A})\hat{q},$   $A = \hat{p}\bar{A}\hat{p}^{-1},$  and  $\bar{A}$  is constant, we have  $V^* = p^*(I - \bar{A})\hat{q} = P\hat{p}(I - \hat{p}^{-1}\bar{A}\hat{p})\hat{p}^{-1}\hat{X} = P(I - A)\hat{X}$  when the output system is fixed. Namely

$$V^* = P(I - A)\hat{X}. \tag{7}$$

Also,  $X^* = \hat{p}^*q = \hat{P}\hat{p}q = \hat{P}X$  and  $T^* = \hat{p}^*\bar{T} = \hat{P}\bar{T} = \hat{P}T$  via (6), that is,

$$X^* = \hat{P}X, \tag{8}$$

$$T^* = \hat{P}T. \tag{9}$$

From  $F = X - TE,$  (8) and (9), we can obtain  $F^* = X^* - T^*E = \hat{P}(X - TE) = \hat{P}F,$  i.e.

$$F^* = \hat{P}F. \tag{10}$$

These formulas will be used in the next sections.

### 2.3. Twelve kinds of special semipositive matrices whose spectral radii are all one

It is useful and convenient for us to define the following twelve kinds of special semipositive matrices whose spectral radii are all

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