Applied Mathematics and Computation 217 (2010) 330–338

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Hopf bifurcation and stability for a differential-algebraic biological economic system ☆

Guodong Zhang, Lulu Zhu, Boshan Chen *

Department of Mathematics, Hubei Normal University, Huangshi 435002, China

ARTICLE INFO

Keywords:
Stability
Hopf bifurcation
Differential-algebraic system
Periodic solution
Harvesting

ABSTRACT

In this paper, we analyze the stability and Hopf bifurcation of the biological economic system based on the new normal form and the Hopf bifurcation theorem. The basic model we consider is owed to a ratio-dependent predator–prey system with harvesting, compared with other researches on dynamics of predator–prey population, this system is described by differential-algebraic equations due to economic factor. Here μ as bifurcation parameter, it is found that periodic solutions arise from stable stationary states when the parameter μ increases close to a certain limit. Finally, numerical simulations illustrate the effectiveness of our results.

1. Introduction

In recent years, mankind is facing the problems about shortage of resource and worsening environment. So there has been rapidly growing interest in the analysis and modelling of biological systems. From the view of human need, the exploitation of biological resources and harvest of population are commonly practiced in the fields of fishery, wildlife and forestry management. The predator–prey system plays an important and fundamental role among the relationships between the biological population. Many authors [1–4] have studied the dynamics of predator–prey models with harvesting, and obtained complex dynamic behavior, such as stability of equilibrium, Hopf bifurcation, limit cycle, Bogdanov–Takens bifurcation and so on. Quite a number of references [5–8] have discussed permanence, extinction and periodic solution of predator–prey models. Most of these discussions on biological models are based on normal systems governed by differential equations or difference equations.

In daily life, economic profit is a very important factor for governments, merchants and even every citizen, so it is necessary to research biological economic systems, which can be described by differential-algebraic equations or differential-difference-algebraic equations. At present, most of differential-algebraic equations can be found in the general power systems [9,10], economic administration [11], mechanical engineering [12] and so on. There are also several biological reports on differential algebraic equations [13,14]. Digital control systems of power grids [15] and biological systems, such as neural networks [16] and genetic networks [17], are typical models mixed with both continuous and discrete time sequences which can mathematically be formulated as differential-difference-algebraic equations. In addition, a lot of fundamental analyzing methods for differential algebraic equations and differential-difference-algebraic equations have been presented, such as local stability [18], optimal control [19] and so on. However, to the best of our knowledge, there are few reports on differential-algebraic equations in biological fields. This paper mainly studies the stability and Hopf bifurcation of a new biological

☆ This work is supported by the Education Department of Hubei Province under Grant (Z200622002).
* Corresponding author.
E-mail address: chenbs@hbnu.edu.cn (B. Chen).

0096-3003/$ - see front matter © 2010 Published by Elsevier Inc.
doi:10.1016/j.amc.2010.05.065

economic system formulated by differential-algebraic equations. In what follows, we introduce the new biological economic system.

The basic model we consider is based on the following ratio-dependent predator–prey system with harvest

$$\begin{cases} \frac{du}{dt} = u(r_1 - \epsilon v), \\ \frac{dv}{dt} = v(r_2 - \theta \frac{v}{u} - \alpha E^*). \end{cases} \tag{1}$$

where r_1, r_2 represent growth rate of the prey and predator, respectively, ϵ, θ and α are positive constants, and u and v can be interpreted as the densities of prey and predator populations at time t, respectively, E^* represents harvesting effort. $\alpha v E^*$ indicates that the harvests for predator population are proportional to their densities at time t, when there is no harvesting was considered by Zhou et al. [20] in detail.

Combining the economic theory of fishery resource [21] proposed by Gordon in 1954, we can obtain a biological economic system expressed by differential algebraic equation

$$\begin{cases} \frac{du}{dt} = u(r_1 - \epsilon v), \\ \frac{dv}{dt} = v(r_2 - \theta \frac{v}{u} - \alpha E^*), \\ 0 = E^*(pv - c) - m. \end{cases} \tag{2}$$

where $p > 0$ is harvesting reward per unit harvesting effort for unit weight of predator, $c > 0$ is harvesting cost per unit harvesting effort for predator, $m \geqslant 0$ is the economic profit per unit harvesting effort.

Substituting these dimensionless variables in system (2),

$$x = \epsilon u, \quad y = \epsilon v, \quad E = \alpha E^*, \quad p_1 = \frac{p}{\epsilon}, \quad \mu = \alpha m.$$

and then obtain the following dimensionless system of differential-algebraic system:

$$\begin{cases} \frac{dx}{dt} = x(r_1 - y), \\ \frac{dy}{dt} = y(r_2 - \theta \frac{y}{x} - E), \\ 0 = E(p_1 y - c) - \mu. \end{cases} \tag{3}$$

For simplicity, let

$$f(Z, E, \mu) = \begin{pmatrix} f_1(Z, E, \mu) \\ f_2(Z, E, \mu) \end{pmatrix} = \begin{pmatrix} x(r_1 - y) \\ y(r_2 - \theta \frac{y}{x} - E) \end{pmatrix}, \quad g(Z, E, \mu) = E(p_1 y - c) - \mu,$$

where $Z = (x, y)^T$, μ is a bifurcation parameter, which will be defined in what follows.

In this paper, we mainly discuss the effects of the economic profit on the dynamics of the system (3) in the region $R^3_+ = \{\chi = (x, y, E) | x \geqslant 0,\ y \geqslant 0,\ E \geqslant 0\}$.

The organization of this paper is as follows. In Section 2, the local stability of the nonnegative equilibrium points are discussed by the corresponding characteristic equation of the system (5). In Section 3, we study the Hopf bifurcation of the positive equilibrium point depending on the parameter μ, based on the normal form and Hopf bifurcation theorem, we derive the formula for determining the properties of Hopf bifurcation of the biological economic system (3). In Section 4, numerical simulations are performed to illustrate the effectiveness of our results. Finally, this paper ends by a brief conclusion.

2. Equilibria and stability analysis of system (3)

From system (3), we can see that there exists an equilibrium in R^3_+ if and only if the equations

$$\begin{cases} x(r_1 - y) = 0, \\ y(r_2 - \theta \frac{y}{x} - E) = 0, \\ E(p_1 y - c) - \mu = 0. \end{cases} \tag{4}$$

It is obvious that Eq. (4) has an only real solutions: $\chi_0 = (x_0,\ y_0,\ E_0) = \left(\frac{\theta r_1}{r_2 - E_0}, r_1, \frac{\mu}{p_1 r_1 - c}\right)$. It should be noted that we only concentrate on the interior equilibrium of the system (3), since the biological meanings of the interior equilibrium imply that prey and predator and harvesting all exist, which are relevant to our study. So in this paper, a simple assumption that the inequality $0 < \mu < r_2(p_1 r_1 - c)$ holds. In order to analyze the local stability of the positive equilibrium point for the system (3), we first use the linear transformation $\chi^T = QN^T$, where

$$N = (X, Y, \overline{E}), \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{p_1 E_0}{p_1 Y_0 - c} & 1 \end{pmatrix}.$$

Then we have $D_\chi g(\chi) Q = (0, 0, p_1 r_1 - c),\ X = x, Y = y, \overline{E} = \frac{p_1 E_0}{p_1 y_0 - c} y + E$, for which system (3) yields