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Simultaneous solution of fuzzy models: an application to economic equilibrium analysis

Ronald R. Yager*

Machine Intelligence Institute, Iona College, New Rochelle, NY 10801, USA

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Abstract

The fuzzy modeling technology, which uses a rule-based description of the relationship between variables, is discussed. We then turn to the problem of the simultaneous solution of relationships involving fuzzy models. We illustrate this problem by considering the equilibrium problem in economics. In this case we have two relationships, one between price and supply and the other between price and demand, and we desire to find the price for which the supply equals the demand. It is shown that if the fuzzy models have antecedent fuzzy sets which are of the trapezoidal–triangular type then a solution technique can be found.

Keywords: Economics; Equilibrium; Fuzzy equations; Simultaneous fuzzy models

1. Introduction

Modern technology makes great use of the idea of relationships between variables. Once having obtained a relationship between two variables one can use this relationship to obtain the value of the dependent variable given knowledge of the independent variable. Thus, if we have a relationship $V = F(U)$ given a value for U we can obtain the value for V. In some situations, rather than knowing the value of the independent variable, we may have another relationship between the variables, $V = G(U)$, and we want to find a solution that simultaneously solves both equations. A prototypical example of this situation is the solution of two linear equations $y = 2x + 3$ and $y = 6x - 9$. Here, we need to solve for both the values of x and y.

Fuzzy systems models [15] have shown themselves to be a very useful tool for the modeling of complex nonlinear relationships between variables. They have found particular use in modeling of intelligent control systems [9,10,19,20]. Using this technology we model a complex relationship with the aid of a rule base in which each rule represents a part of the whole relationship. In this framework our relationship is a set of rules of the form *if U is* A_i *then V is* B_i where A_i and B_i are fuzzy subsets. As indicated above once having obtained a fuzzy model a typical usage of this model involves the determination of the output variable for a given value of the input variable. This is accomplished by using the fuzzy inference algorithm. In this work we consider an extension of the usage of the fuzzy systems modeling technology to the case in which, rather than knowing the value of the input variable, we have some other relationship between the variables, which may be another fuzzy model, and desire to find

* Tel.: +1 212 249 2047; fax: +1 212 249 1689.
E-mail address: yager@panix.com (R.R. Yager).

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the simultaneous solution of these two models. We illustrate this situation by considering the equilibrium problem in economics [8]. In this case we have two relationships, one between price and supply and the other between price and demand, and we desire to find the price for which the supply equals the demand.

2. Fuzzy systems modeling

Fuzzy modeling, which is rooted in the pioneering work of L.A. Zadeh on the theory of approximate reasoning [18,21] found its first application in the work by Mamdani and his colleagues [5–7] on the design of a controller for a cement kiln. Sugeno [11–13] provided an important extension of the original paradigm for fuzzy modeling. While the original work by Mamdani and his colleagues was in the design of an intelligent controller fuzzy systems modeling can be more generally seen as a technique for the modeling of complex functional relationships [4]. This ability to model complex non-linear relationships allows it to be used in many diverse disciplines. In the following, we shall briefly review the fuzzy modeling technology.

Assume U and V are two variables which are related by some functional relationship, [1] $V = f(U)$, and which respectively take their values in the spaces X and Y, typically, subsets of the real line. In fuzzy modeling this relationship is approximated by a rule base which describes the known relationship between U and V. Typical of these rules is a statement of the form

if U is ***close to*** **50** *then V is* ***large.***

More formally, we represent the relationship between U and V by a collection of n rules of the form

$$\text{if } U \text{ is } A_i \text{ then } V \text{ is } B_i. \tag{I}$$

In this formulation the A_i's and B_i's are fuzzy subsets, corresponding to linguistic concepts, defined over the spaces X and Y, respectively. Using the fuzzy modeling approach we are essentially partitioning the variable U into fuzzy regions in which we have a good approximation of the dependent variable V. Fig. 1 provides a view of the fuzzy modeling process.

[1] For ease of explanation we assume only one independent variable, U, more generally we can have multiple independent variables.

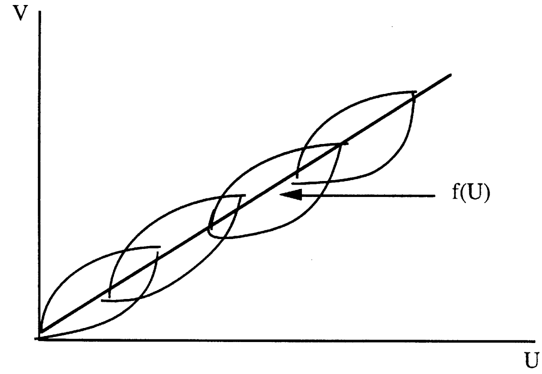

Fig. 1. View of fuzzy partitioning.

In this view we are modeling the underlying function by a union of fuzzy patches each of which is of the form $A_i \times B_i$.

The use of a fuzzy model to determine the value of V for a given input value, x^* of U is based upon the use of a weighted combination of the rule consequents. The weighting in this process is determined by the relevancy of the rule to the given input. This degree of relevancy is often called the firing level of the rule. This process of determining the output for a given input is called *fuzzy inference*. The algorithm used to implement this fuzzy inference process is described in the following. Assume we have a fuzzy model of the type expressed in Eq. (I). For a given value of U, $U = x^*$, we obtain the value y^* for V using the following algorithm:

Fuzzy inference algorithm

(1) Find the degree, τ_i, to which rule i fires,

$$\tau_i = A_i(x^*) \text{ (the membership grade of } x^* \text{ in } A_i).$$

(2) Find the effective output of rule i as the fuzzy subset E_i whose membership grade is determined as

$$E_i(y) = T(\tau_i, B_i(y)).$$

(3) Find the overall model output as the fuzzy subset E which is the union of the E_i. The membership grade of E is determined as

$$E(y) = S(E_1(y), E_2(y), \ldots, E_n(y)).$$

(4) Defuzzify E to obtain a crisp output value, y^*.

The operation $T(\tau_i, B_i(y))$ is normally implemented by a t-norm operator [3], a generalized multivalued logic *and* operator. One characterizing property of the