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Existence of Solutions for Abstract Economic Equilibrium Problems and Algorithm

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Abstract—In this paper, we study a new kind of vector equilibrium problem and we proved the existence of solutions for this kind of equilibrium problem by using the section theorem and Fan KKM theorem. Then we generalize this kind of vector equilibrium problem to a more general case. Equilibrium problems with lower and upper bounds was an open problem proposed by Isac, Sehgal and Singh in 1999. J.Li, O.Chadli, Y.Chiang and J.C.Yao, Zhang Cong-jun derived some results of the equilibrium problems with lower and upper bounds under certain conditions. In this paper we derive more results of this open problem under certain conditions, constructs an iteration algorithm, and discuses the convergence of the algorithm.

Keywords—Economic equilibrium problem, Vector equilibrium problem, Equilibrium problems with lower and upper bounds, Section theorem, iteration algorithm.

1. INTRODUCTION

The general equilibrium theory was proposed by Walras at the end of 19 centuries. It regards economic analysis in complete competition as the main contents. It includes the economic system which was composed of the consumer and producer and large quantity of the money and ware in his model. His theory is, if this time exist appropriate price system, under this price system each main body act as acceptors, then the consumer can get the biggest benefits, producer can acquire the biggest profits, and make the money and ware attain the complete competition equilibrium

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state. This conclusion was called the Walras compete competition equilibrium exist theorem. It was proposed by Walras, but the strict proof was gaven by the mathematician Debreu after half a century in 1952. Therefore Debreu won the Nobel economics prize in 1983, Debreu proved the Walras compete competition equilibrium exist theorem by using fixed-point theorem of set-valued mapping. Henceforth, many economists start study different ware spaces the price system influenced by different factor, discuss the existence of equilibrium point, because of the factor which influence the price more and more, the ware space also extend to infinite dimension from finite dimension. Many mathematicians start to prove it under what conditions the equilibrium point exist, and discuss the algorithm of the equilibrium point, studying the convergence of the algorithm.

At the foundation of predecessors,in this paper, we study a new kind of vector equilibrium problem. In the third section, we proved the existence of solutions for this kind of equilibrium problem by using the section theorem and KKM theorem these two tools. Then we generalize this kind of vector equilibrium problem to a more general case. In the following section, we study equilibrium problems with lower and upper bounds. In the fourth section, we derived a new result of the open problem on certain conditions. In the fifth section, we constructs an iteration algorithm. Then we discuses the convergence of the algorithm in the sixth section.

2. PRELIMINARIES

DEFINITION 2.1. *Let $G : X \to 2^Y$ be a set-valued mapping, W be a open set of Y. G is called upper semi-continuous at x_0, if for open set W containing $G(x_0)$, there exist a neighborhood U of x_0, such that for every $x \in U$, we have $G(x) \subset W$.*

DEFINITION 2.2. *Let $F : X \times X \to 2^Y$ be a set-valued mapping. F is called convex related to second variable, if for any $y_1, y_2 \in X$, $\lambda \in (0,1)$*

$$F(x, \lambda y_1 + (1-\lambda)y_2) \subset \lambda F(x, y_1) + (1-\lambda)F(x, y_2).$$

DEFINITION 2.3. *Let $F : K \to 2^Y$ be a set-valued mapping. F is called KKM-map, if for every finite subset $\{x_1, x_2, \dots, x_n\}$ of K, we have*

$$\mathrm{co}\,\{x_1, x_2, \dots, x_n\} \subset \bigcup_{i=1}^{n} F(x_i).$$

DEFINITION 2.4. *Let $f : K \to K$ be a single-valued mapping. f is called affine mapping, if for any$x, y \in K$, and $t \in [0,1]$,*

$$f(tx + (1-t)y) = tf(x) + (1-t)f(y).$$

The following lemma is Ky Fan section theorem, see [1].

LEMMA 2.1. *Let X be a real Hausdorff topological vector space and let K be a nonempty compact convex set of X. Let $A \subset K \times K$, such that for every $x \in K$, $(x,x) \in A$. If one of the following conditions holds:*

1. *$\forall x \in K$, $A_x = \{y \in K, (x,y) \notin A\}$ is convex set or empty set; $\forall y \in K$, $A_y = \{x \in K, (x,y) \in A\}$ is closed;*
2. *$\forall x \in K$, $A_x = \{y \in K, (x,y) \in A\}$ is closed; $\forall y \in K$, $A_y = \{x \in K, (x,y) \notin A\}$ is convex set or empty set.*

Then there exists $x_0 \in K$, such that $\{x_0\} \times K \subset A$.

The following lemma is generalized section theorem, see [2].