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Adaptive control of manufacturing systems with incomplete information about demand and inventory

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Abstract: Optimization of failure-prone manufacturing systems with uncertainty in demand and inventory levels are considered. Conventional approach to production optimization of manufacturing systems requires precise knowledge of demand and inventory. In case of unknown demand and imprecise inventory the online estimation is needed in order to implement optimal policies. We propose in this work a novel methodology for monitoring and online assessment of demand and inventory levels based on the state estimators in the case of precisely known inventory and on Kalman filtering in the case of uncertainty in inventory information. The proposed approach allows to continuously monitor the uncertain states of the system and provides converging estimates of demand and inventory levels (when they are not precisely measured). The cases of constant, slow-varying and randomly-varying demand are considered. Obtained estimates are shown not to be affected by the discontinuous changes in production capacity, that are due to random failure-repair processes. The proposed methodology is described in detail for the case one-machine-one-product system and the possible extensions to the case of hybrid manufacturing-remanufacturing systems with uncertain demand and return are outlined.

Keywords: Manufacturing system, failure-repair, demand, inventory, uncertainty, forecast, state estimator, Kalman filter.

1. INTRODUCTION

Manufacturing systems are usually functioning in the environments with several uncertainties. Optimizing the system behavior while accounting for these uncertainties is practically important but theoretically challenging. Generic approaches to the analysis of the systems with only partially observed inventory characteristics are discussed in Kang and Gershwin (2005) and Sethi and Shi (2013). The reasons for which the observed inventory level may differ from the actual one are numerous: spoilage, theft or misplacement of the products, transaction errors or information delay, etc. In Kang and Gershwin (2005) the sensitivity of the system performance to the inventory record errors are studied with particular attention to the systems operating under (Q,R)-policy. In Sethi and Shi (2013) various methods for compensating the uncertainty impact were discussed. Existing approaches to incomplete information about inventory were reviewed and new solution methods were proposed mainly in the framework of inventory management. A practically important class of the systems with partially observed inventory (so called "zero balance walk" - model) is investigated in Bensoussan et al (2007), where the rigorous analysis of optimality conditions (Bellman equations) is performed and a feedback control policy is described.

Additionally to having the incomplete information about inventory levels, manufacturing systems are often functioning under uncertain demands. A usual assumption that the demand is a known constant is a convenient but barely justifiable simplification. In practice, the demand has to be forecasted and inevitable forecasting errors propagate through the system and affect all subsequent managerial decisions. The issues related to demand forecasting have recently gained a substantial interest of scientific community. In Tratar (2015) various forecasting methods are compared in order assess their suitability for the case of noisy demand. The influence of demand forecasting errors on production and maintenance optimization and on inventory control is investigated in Hajej et al (2014) and in DoRego and deMosquita (2015(@) respectively. The deep impact of demand forecasting on the safety stock level that has been often neglected in the literature is discussed in Prak et al (2017).

For the systems that use remanufacturing in their production process the effect of uncertainty in the demand and (especially) in the return level is more pronounced (Govindan et al (2015)), and currently attracts the increasing interest of the researchers. In particular, in Zang and Hu (2014) authors describe the decisions about the acquisition price and production rates that the company has to set while facing uncertainty in the demand and return levels.

2. PROBLEM FORMULATION AND SYSTEMS UNDER STUDY

In order to demonstrate our proposed approach we consider the simplest possible model manufacturing system,

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namely a one-machine-one-product system described in Akela and Kumar (1986), where analytical solution for production optimization problem was given.

2.1 Problem statement

The machine is subject to (random) failures followed by (random) repairs. The times between successive failures and the repair times are exponentially distributed with rates p and r respectively. We denote by ξ the binary variable corresponding to the random state of the machine ($\xi = 1$ when the machine is up and $\xi = 0$, when it is down). Transitions between the state are conventionally described by the state transition matrix

$$G = \{q_{ij}\} = \begin{pmatrix} -p & p \\ r & -r \end{pmatrix} \quad (1)$$

Let us denote by x the serviceable inventory, by z its measured value, by d - the demand rate, by u and U - the production and maximal production rates respectively. The evolution of the system can be described as follows

$$\begin{aligned} \dot{x} &= u - d \\ \dot{d} &= 0 \\ z &= x + v \end{aligned} \quad (2)$$

Additionally to conventionally used first equation we have added to the model the equations that describes the demand behavior (second line) and inventory measurement (third line).

2.2 Proposed methodology

In contrast to the standard assumptions that the instantaneous demand and inventory levels are precisely known we suppose that one of them or both are uncertain. Namely, we consider the cases of (a) perfectly known inventory level and (b) inventory level being subject to uncertainty. In case (a) we further distinguish the sub-cases of (a.1) constant and (a.2) variable demand. In case (b) we consider inventory measurements corrupted by the white additive noise, and suppose the demand being a sum of a constant and a first order Markov process.

For the case of perfectly measured inventory and unknown demand (constant or varying), we propose an estimation procedure based on the use of *state observers*. For the case of varying demand we suppose that the model describing the demand evolutions is available. We construct the demand estimate that converges asymptotically to the exact (unknown) demand and this estimate can be used (instead of the unknown demand) for computing optimal policy .

For the case of the errors in the inventory measurement channel we propose the estimation procedure that uses *Kalman filter*. We consider in this case the demand dynamics as being random as well. Manufacturing systems with random demand were considered previously (see e.g Ouaret et al (2013) and references therein), however the *guaranteed* solution was mostly under study, while we explore the possibility of designing an adaptive strategy based on the constructed estimates.

Our approach is inspired by the so-called *separation principle* from the feedback control theory. We first compute the estimates of the demand and inventory levels using the Kalman-filter-based technique, then replace the unknown inventory and demand levels by the obtained estimates and use them in the Hamilton-Jacobi-Bellman (HJB) equations to obtain the (sub)optimal production control policy. Preliminary results in that direction were presented at the 11-th International Conference on Modeling Optimizagtion and simulation (MOSIM'16).

3. DEMAND ESTIMATION FROM THE KNOWN INVENTORY DYNAMICS

We suppose that the inventory level can be directly measured, and the production capacity (at each moment of time) is precisely known. That means that when the machine switches to failure mode (and therefore its capacity falls to zero level) - it becomes immediately known. A dynamical system, called state observer is constructed, that takes as inputs the current production level, the measured inventory level, and its output is the estimated demand level. The estimates provided by the sate observer are known to converge under some conditions to the actual (unknown, variable) demand level.

3.1 Constant demand

Let us consider the system model (2) with constant demand and precise inventory measurements. We therefore have:

$$y = x(t) \quad (3)$$

To define the observer dynamics, we first define the observer gains that allow to place the observer poles into the desired location. For our case we chose the double pole located at the point $\lambda_1 = \lambda_2 = -2$ on the complex plane. To do that we set the degree of stability μ ($\mu = 2$ is chosen in our case), and then proceed with defining the gains as follows:

$$g_1 = -2\mu; \; g_2 = \mu^2 \quad (4)$$

This definition leads to a second order observer:

$$\begin{aligned} \dot{\tilde{x}} &= u - \tilde{d} + g_1(\tilde{x} - y) \\ \dot{\tilde{d}} &= g_2(\tilde{x} - y) \end{aligned} \quad (5)$$

Corresponding error dynamics is of second order, thus the demand estimation error converges exponentially with the rate 2 (bounded by $\exp -2t$). The described procedure works well for a constant demand and provides the estimate converging exponentially fast to the actual *unknown* demand.

The behavior of constructed estimates as well as corresponding inventory dynamics are illustrated in figure 2. It is important to emphasize that the discrete stochastic jumps in the production $u(t)$ due to failure-repair random perturbations do not affect the estimation process. That is because the production (even affected by the failures) is known, and as it is being integrated into the estimation procedure, the resulting error dynamics is invariant to such perturbations.

In a particular case illustrated in figure 2 the following system parameters are used: $MTTF = 0.1$ ($p =$