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A simple unit root test against asymmetric STAR nonlinearity with an application to real exchange rates in Nordic countries

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ABSTRACT

Existing tests of the unit root hypothesis against the alternative hypothesis of exponential smooth transition autoregressive (ESTAR) nonlinearity implicitly assume symmetry under the alternative. This paper proposes a simple unit root test against the alternative of symmetric *or* asymmetric ESTAR nonlinearity. In the event that the unit root hypothesis is rejected, a simple test of symmetric versus asymmetric ESTAR nonlinearity is also proposed. The asymptotic distributions of the test statistics are straightforward to establish and finite-sample performance is studied with Monte Carlo simulations. An empirical application involving the real exchange rates of four Nordic countries against the U.S. dollar illustrates the usefulness of the new tests.

1. Introduction

The exponential smooth transition autoregressive (ESTAR) time series model has proved to be popular in economics for the analysis of time series data on the relative prices of equivalent goods or assets, such as data on real exchange rates. The presence of transactions costs suggests that while large deviations of real exchange rates from their equilibrium values will be corrected by arbitrage, small deviations may not be corrected, and the globally stationary ESTAR model with a unit root central regime is able to capture this type of nonlinearity; see for example Michael et al. (1997), Baum et al. (2001), and Taylor et al. (2001). A number of tests of the unit root hypothesis against stationary ESTAR nonlinearity have recently been proposed; see for example Kapetanios et al. (2003) and Park and Shintani (2005). Although the ESTAR models used by these authors differ fundamentally to conventional linear autoregressive (AR) models as they allow the AR parameters to vary as a function of a previous lag of the series, an implicit assumption of both linear AR *and* nonlinear ESTAR model is that at any point in time the strength of any mean reversion is symmetric, in the sense that at any point in time the AR parameters are identical for positive deviations of the series from its attractor as for negative deviations of the same proportionate amount.

In some empirical applications the assumption of symmetric mean reversion (linear or nonlinear) is not overly restrictive; however in other empirical applications one might expect *asymmetry* in the adjustment of the process towards its equilibrium. For example, in the case of real exchange rates, one might expect asymmetric adjustment if domestic or foreign policymakers behave asymmetrically in response to appreciations and depreciations of the same proportionate amount. Sollis et al. (2002) find evidence suggesting that *asymmetric* nonlinear mean reversion is an important feature of data on real exchange rates against the U.S. dollar. Sollis et al. (2002) propose a unit root test that allows for a form of asymmetric STAR nonlinearity under the alternative hypothesis, related to the ESTAR form. However, their tests are computationally expensive requiring the estimation of asymmetric nonlinear models to compute the test statistics. Furthermore, due to the presence of unidentified parameters under the null hypothesis being tested, they are not able to derive the asymptotic distributions of the test statistics proposed.

In this paper we propose an extended version of the ESTAR model and develop a simple test of the unit root null from this extended model that allows for symmetric *or* asymmetric stationary ESTAR nonlinearity under the alternative. In the event that the unit root hypothesis is rejected, the specification proposed can also be used to distinguish between symmetric and asymmetric ESTAR nonlinearity using a standard *F*-test. The non-standard asymptotic distribution of the proposed unit root test is established and Monte Carlo simulations demonstrate that relative to competing tests the proposed unit root test has good finite-sample properties. An empirical application to the real exchange rates of four Nordic countries against the U.S. dollar further illustrates the usefulness of the new tests.

The rest of the paper is set out as follows: Section 2 very briefly reviews some existing ESTAR unit root tests before introducing the

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extended ESTAR model and the unit root tests derived from this model. Section 3 discusses the Monte Carlo results. Section 4 briefly discusses the empirical application. Section 5 concludes.

2. Unit root tests against symmetric and asymmetric ESTAR nonlinearity

2.1. Existing unit root tests against ESTAR nonlinearity

Since the Sollis et al. (2002) unit root tests do not strictly test against the ESTAR alternative (but a related form of nonlinearity), for brevity we will compare the test developed here only with the more recent Kapetanios et al. (2003) and Park and Shintani (2005) tests. Kapetanios et al. (2003) employ the following ESTAR specification:

$$\Delta y_t = G_t(\gamma, y_{t-d})\rho y_{t-1} + \varepsilon_t \quad (1)$$

$$G_t(\gamma, y_{t-d}) = 1 - \exp\left(-\gamma\left(y_{t-d}^2\right)\right) \quad \gamma \geq 0 \quad (2)$$

where ε_t is an independently and identically distributed (iid) sequence with zero mean and constant variance, and $d=1$ is assumed. Assuming $\rho<0$ and $\gamma>0$, for small deviations from its attractor in period $t-1$ (the attractor here is the mean zero, but can be a non-zero mean or deterministic trend — see below), the series in period t, y_t, is a near-unit root process. For the limiting case $y_{t-1}\rightarrow 0$, y_t approaches a random walk. However, for large deviations from its attractor in period $t-1$, y_t is a stationary AR(1) process with AR parameter $(1+\rho)$. Despite having a unit root central regime, y_t can be thought of as being globally stationary (a simple proof of asymptotic stationarity is given in Kapetanios et al., 2003, utilising the drift condition of Tweedie, 1975; see also Tong, 1990, Appendix 1). This model can be extended to deal with a non-zero mean and a deterministic trend by replacing y_t with $y_t^*=y_t-\hat{\mu}$ or $y_t^*=y_t-\hat{\alpha}_1-\hat{\alpha}_2 t$, where $E(y_t)=\mu$, and $\hat{\mu}$, $\hat{\alpha}_1$, $\hat{\alpha}_2$ are estimates of the relevant population parameters obtained by least squares (LS) regressions prior to the ESTAR model being estimated.

If $\gamma=0$ in Eq. (1), then y_t is nonstationary and contains a fixed unit root. Hence the relevant null hypothesis is H_0:$\gamma=0$. However, under this null hypothesis the parameter ρ is unidentified, and so testing this null using conventional methods is not feasible. Following Luukonen et al. (1988), Kapetanios et al. (2003) deal with this identification problem by replacing $G_t(\gamma, y_{t-d})$ $(d=1)$ with a first-order Taylor expansion around $\gamma=0$. On re-arranging, this gives the auxiliary model,

$$\Delta y_t = \delta y_{t-1}^3 + \eta_t \quad (3)$$

where $\eta_t=\varepsilon_t+R_t$, with R_t denoting the remainder from the Taylor expansion and $\delta=\gamma\rho$. The null hypothesis is H_0:$\delta=0$ in Eq. (3). Kapetanios et al. (2003) propose testing this null hypothesis using the t-statistic:

$$t_{\mathrm{NL}} = \frac{\hat{\delta}}{\mathrm{se}\left(\hat{\delta}\right)} \quad (4)$$

where $\hat{\delta}$ is the LS estimate of δ and se$(\hat{\delta})$ is the LS standard error. Higher-order dynamics can be dealt with in the usual way — by augmenting Eq. (1) with lags of Δy_t. The relevant auxiliary model can be written:

$$\Delta y_t = \delta y_{t-1}^3 + \sum_{i=1}^{k} \kappa_i \Delta y_{t-i} + \eta_t \quad (5)$$

In Monte Carlo simulations Kapetanios et al. (2003) show that the test statistic (Eq. (4)) has good finite-sample properties. They demonstrate the usefulness of this test with empirical applications to real exchange rates and real interest rates. Hereafter, let t_{NL}, $t_{\mathrm{NL},\mu}$ and $t_{\mathrm{NL},t}$ denote the Kapetanios et al. (2003) tests for the zero mean, non-zero mean and deterministic trend cases respectively.

Park and Shintani (2005) take an alternative approach to Kapetanios et al. (2003). Rather than having to deal with the problem of unidentified parameters under the null by taking a Taylor expansion of the underlying ESTAR model, they circumvent this problem by testing the unit root hypothesis using the infimum of the t-statistic on $\hat{\rho}$ in the original ESTAR model (Eq. (1)), over a grid of values for the speed of transition parameter γ. Park and Shintani (2005) develop an encompassing asymptotic theory that allows for various different transition functions including but not limited to the exponential function. Whilst they do not consider the asymmetric ESTAR functional form proposed in this paper, it is likely that their asymptotic theory will also apply to this case. However, this point is not pursued here, and as a competitor to the test developed in this paper we employ the Park and Shintani (2005) unit root test against *symmetric* ESTAR nonlinearity, which hereafter will be labelled inf t_μ (the subscript μ indicating that a non-zero mean is allowed).

2.2. An extended ESTAR model

We propose testing the unit root hypothesis using an extended version of the ESTAR model that allows for symmetric or asymmetric nonlinear adjustment under the alternative hypothesis to a unit root. The extended ESTAR model developed here, referred to as an asymmetric ESTAR (AESTAR) model, employs both an exponential function and a logistic function as follows (like Kapetanios et al., 2003, we assume throughout that the transition variable is y_{t-1}):

$$\Delta y_t = G_t(\gamma_1, y_{t-1})\{S_t(\gamma_2, y_{t-1})\rho_1 + (1-S_t(\gamma_2, y_{t-1}))\rho_2\}y_{t-1} + \varepsilon_t \quad (6)$$

$$G_t(\gamma_1, y_{t-1}) = 1-\exp\left(-\gamma_1\left(y_{t-1}^2\right)\right) \quad \gamma_1 \geq 0 \quad (7)$$

$$S_t(\gamma_2, y_{t-1}) = [1+\exp(-\gamma_2 y_{t-1})]^{-1} \quad \gamma_2 \geq 0 \quad (8)$$

where $\varepsilon_t \sim \mathrm{iid}(0, \sigma^2)$. A similar, but not identical type of AESTAR model has been used by Anderson (1997) and Siliverstovs (2005). These authors do not consider the issue of unit root testing. Assuming for the purposes of exposition that $\gamma_1>0$ and $\gamma_2\rightarrow\infty$, as y_{t-1} moves from zero towards $-\infty$ then since $S_t(\gamma_2, y_{t-1})\rightarrow 0$, an ESTAR transition occurs between the central regime model,

$$\Delta y_t = \varepsilon_t \quad (9)$$

and the outer-regime model,

$$\Delta y_t = \rho_2 y_{t-1} + \varepsilon_t \quad (10)$$

with γ_1 determining the speed of the transition. As y_{t-1} moves from zero towards ∞, then since $S_t(\gamma_2, y_{t-1})\rightarrow 1$ an ESTAR transition occurs between the central regime model,

$$\Delta y_t = \varepsilon_t \quad (11)$$

and the outer-regime model,

$$\Delta y_t = \rho_1 y_{t-1} + \varepsilon_t \quad (12)$$

with γ_1 determining the speed of the transition. Clearly if $\rho_1\neq\rho_2$, the autoregressive adjustment is asymmetric either side of the attractor (in this case the attractor is zero). Global stationarity requires $\rho_1<0$, $\rho_2<0$, $\gamma_1>0$.[1] Note that Eq. (6) nests the symmetric ESTAR specification of Kapetanios et al. (2003), since if $\rho_1=\rho_2=\rho$, Eq. (6) is equivalent to Eq. (1).

[1] This is straightforward to prove using the drift condition of Tweedie (1975), as in Kapetanios et al. (2003) for the symmetric case.