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IFAC PapersOnLine 50-1 (2017) 10172–10177

On Inventory Control For Perishable Inventory Systems Subject To Uncertainties On Customer Demands

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Abstract:
This paper deals with the inventory controller design for constrained production systems subject to uncertainties on the customer demands. The case study focuses on the inventory regulation problem in production systems where contain perishable finite products. Such systems are characterized by the presence of delays due to production processes, and constraints from the instantaneous inventory level, production level and the finite capacities of stocks. To do that, we propose a management strategy based on the inventory control, using either a linear control law or a bang-bang control. A design method is proposed to determine the parameters of an admissible control law. The design method is based on the invariance principle, and our proof is based on the exact identification of the admissible region on the space of the parameters of the system and the control specifications.

Keywords: Inventory control design, Perishable finite products, Time-delay systems, predictor-feedback structure, $\mathcal{D}$-invariance properties.

1. INTRODUCTION

In this paper, we are interested on the inventory regulation problem in perishable inventory systems that must satisfy the customer demand the we suppose unknown but bounded by defined values. Also, the production system is characterized by the presence of delay due to the process time, and the positivity constraints due to the specifications of the system, such as production and storage capacities. Another characteristic of the system is the presence of a perishability factor which is related to the stock. The main difficulty is developing control laws for perishable inventories stems from the necessity of conducting an exact analysis of product lifetimes. The difficulty to obtain a robust control is more complicated in the situation when the customer demand is subject to significant uncertainty and the inventories are replenished with non-negligible delay. Our objective is to maintain high service level without interruption and at the same time to minimize the cost inventory. For that, we must define a control law which is useful to satisfy the demand during procurement latency but also to compensate the stock deterioration in that time.

In this paper, we carried out a study on the inventory control problem of logistical systems subject to delays and perishable finite products using an approach based on control theory. Those systems are subject to positivity and saturation constraints mainly related to the physical characteristics of the processes and should be taken into account in the modelling of systems and the design of control laws to stabilize them with respect to the delay. We have introduced the basic model for each node of a logistic system, Which is a delay system on the input. This delay is interpreted as the duration of production operations. The logistic system considered must satisfy a customer demand while respecting constraints induced by the positivity of the variables and the finite capacity of the processes.

We proposed two permissible command structures: the first is an affine law and the second is a bang-bang type law, which serve to stabilize the logistical system under consideration, whilst respecting the constraints of saturation and positivity, and for any unknown but bounded demand $d(t)$. We studied a delayed logistic system for zero initial conditions, that is to say without taking into account its dynamics between the instants 0 and $theta$. We have obtained necessary conditions for the existence of the admissible laws according to the intrinsic parameters of the system, such as $sigma$, $theta$, etc.

This article is organised as follows. The second section is dedicated to the problem statement and the characteristics of the considered problem. description of the problem. In the third section, some backgrounds about the $\mathcal{D}$-invariance properties are briefly described. In section 4, the inventory control problem with perishable finite products and under unknown demand is developed. The section 5 is dedicated to the controller design, where the conditions of the existence of the controller are given. The paper

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.1765

concludes with some discussions, as well as the directions of the future work.

2. PROBLEM STATEMENT

2.1 Model Description

In this study, we consider an elementary production system composed of a supplying unit and a storage one. The supplying unit is characterized by a supplying order rate denoted $u(t)$, which is limited by a minimum value denotes u_{min} and a maximum supplying order rate denoted u_{max}. Furthermore, the production system is characterizer by a delay θ, $\theta \geq 0$, which corresponds to the time needed to complete the finite products. The storage unit is characterized by the inventory level denoted $y(t)$ bounded by a minimum and maximum storage capacity denoted, respectively, y_{min} and y_{max}. In this work, the customer demand denoted $d(t)$ is supposed to be unknown but assumed to be bounded by two values denoted d_{min} and d_{max}, known in advance, and corresponding to, respectively, minimum and maximum demand rates.

This production system is as time-delay system with perishable products defined by an expiration rate named σ. Indeed, $u(t)$ is the control input, $d(t)$ is an external perturbation, and $y(t)$ is the output to be controlled. The generic model for the inventory level dynamics is then described by the following first order delayed equation.

$$\dot{y}(t) = \begin{cases} -\sigma y(t) + u(t-\theta) - d(t) & \text{for } t \geq \theta \,, \\ -\sigma y(t) - d(t) & \text{for } 0 \leq t < \theta \,. \end{cases} \tag{1}$$

$u(t)$, $y(t)$ and $d(t)$ are non negative variables and they represent, respectively, the production level, the inventory level and the customer demand. In this paper, the study of the production system is focused on the horizon time t, $t \geq \theta$.

2.2 Constraints And Objective

The objective of this study consists first to define necessary and sufficient conditions of existence of an admissible control law $u(t)$ in order to fulfil the demand $d(t)$ taking into account the different constraints. related to supplying units and inventories which are limited resources, and they can take only non-negative values. These constraints are formulates as follows.

The controller should be designed such that, for all $t \geq 0$:

$$y_{min} \leq y(t) \leq y_{max} \,, \tag{2}$$

with

$$u_{min} \leq u(t) \leq u_{max} \,, \tag{3}$$

and for every demand function satisfying

$$d_{min} \leq d(t) \leq d_{max} \,. \tag{4}$$

3. PROPOSED CONTROL STRATEGY

3.1 Prediction and invariance

As developed in (Moussaoui, 2014) and extended in (Abbou et al.,, 2015a) and (Abbou *et al.*, 2015b), our proposed approach to control systems with delayed inputs is based on a **predictor based feedback structure**. This structure permits to stabilize the system and to compensate the delay effects present in the loop. The specifications of the production system are introduced as constraints imposed to the controller, so as to forbid any overruns on the production rates or on the inventory levels, which can cause the saturation of the production unit. The role of the controller is then to keep the production rate and so, the inventory level, as far as possible within their limits.

Using the feedback-predictor structure, also known as model reduction or Arstein reduction (Artstein, 1982), the basic idea of state prediction is to compensate the time delay θ by generating a control law that enables one to directly use the corresponding delay-free system, thanks to the prediction expressed by

$$z(t) = \mathrm{e}^{-\sigma\theta} y(t) + \int_{t-\theta}^{t} \mathrm{e}^{-\sigma(t-\tau)} u(\tau) d\tau \,. \tag{5}$$

By time derivation of the equation (5), one can see that the resulting system

$$\dot{z}(t) = -\sigma z(t) + u(t) - \mathrm{e}^{-\sigma\theta} d(t) \,. \tag{6}$$

is free-delay. Then we can applicate the invariance theory explained in the next paragraph.

3.2 $\mathcal{D}$-invariance concept

We consider a function f defining a system $\dot{x}(t) = f(x(t)) - d(t)$. The interval $\mathcal{Z}$ defined as $\mathcal{Z} = [z_{min}, z_{max}]$ is $\mathcal{D}$-invariant for this system, with $\mathcal{D} = [d_{min}, d_{max}]$, if and only if the following conditions are fulfilled.

$$f(z_{min}, d_{max}) \geq 0 \tag{7}$$

$$f(z_{max}, d_{min}) \leq 0 \tag{8}$$

If we consider the system defined by the equation (6), and we applicate the $\mathcal{D}$-invariance results, we deduce the following expressions for the maximum and the minimum values of $z(t)$, for $z(t) = z_{min}$ and for $z(t) = z_{max}$.

If we consider the system defined by the equation (6), and we applicate the $\mathcal{D}$-invariance results, we deduce the following expressions for the maximum and the minimum values of $z(t)$, for $z(t) = z_{min}$ and for $z(t) = z_{max}$

For $z(t) = z_{min}$, the expression is

$$-\sigma z_{min} + u(t) - \mathrm{e}^{-\sigma\theta} d_{max} \geq 0 \tag{9}$$

and for $z(t) = z_{max}$, we get

$$-\sigma z_{max} + u(t) - \mathrm{e}^{-\sigma\theta} d_{min} \leq 0 \tag{10}$$

3.3 Types of control laws

We can consider two forms of control laws that allow the stability of the system (1) in closed loop taking into account positivity and saturation constraints (2) et (3). For each form of the control law, we consider two values u_1 and u_2 which fulfil the constraint (3) expressed by

$$u_1 \in [u_{min}, u_{max}] \tag{11}$$

and

$$u_2 \in [u_{min}, u_{max}] \,. \tag{12}$$