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# Idling Policies for Periodic Review Inventory Control ⋆

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**Abstract:** This paper presents a study on idling production policies in the context of periodic review inventory control. The usefulness of these policies will be demonstrated by means of two different numerical experiments from which we will be able extract some interesting managerial insights, as well some structural properties of the solutions. In the first experiment, the stabilization properties of these policies will be illustrated by means of the stabilization of the Lu&Kumar network. In the second experiment, we will present a stable system for which idling policies achieved a lower operational cost when faced with their non-idling counterparts.



*Keywords:* Inventory Control, Idling Policies, Infinitesimal Perturbation Analysis, Simulation Based Optimization.

## 1. INTRODUCTION

In the context of industrial applications, inventory control policies address the problem of splitting a finite amount of capacity among a set of different products that need processing. In the context of periodic review inventory control, the base stock policy is known to be optimal for single product, single machine systems (capacitated or uncapacitated). Also, for uncapacitated machine flow lines it is known that the Multi-Echelon Base Stock policies – MEBSP – are optimal, (Clark and Scarf, 1960). However, when machines are capacitated, little is known about the structure of the optimal inventory policy, except that the optimal final product inventory levels are bounded. Despite knowing this, there are no closed form results when it comes to determining the optimal policy nor these bound values. When using MEBSP one still has to address how to split capacity among the different products when their requirements are above the capacity value. Many authors have used variations of the MEBSP proposing different dynamic rules to address the capacity split. Examples are Priority, Linear Scaling, Equalize Shortfall, and Weighted Equalize Shortfall, among others (See (Janakiraman et al., 2009)). One particular feature of all these variations is the fact that they all are non-idling. The production decisions for a given decision period are solely limited by feeding inventory and machine capacity.

We claim and show that tighter limits to the production decisions may be needed to guarantee stability and to achieve better performances. The main objective of the work described here is to study idling policies for periodic review inventory control and to establish a framework where they can be compared with the non-idling approaches.

This paper presents an example of a stable system where these tighter production bounds are used to achieve performance improvements. The stabilization of an unstable network similar to the one described in Lu and Kumar (1991) using these production bounds to induce idleness will also be presented.

This paper is organized as follows. We start by introducing the theoretical model as well as its dynamic equations, Section 2. The Infinitesimal Perturbation Analysis as well as its applicability will be discussed in Section 3. Section 4 will present the stabilization of a network similar to the one of Lu&Kumar using the proposed policies. In Section 5 we will present an experiment where, using IMEBSP, we were able to improve operational cost over their non-idling counterparts. The paper ends, in Section 6, with some conclusions and references to future work.

## 2. THEORETICAL MODEL

In (Bispo, 1997), the author presents a framework to study MEBSP, in the context of periodic review inventory control, for simple re-entrant systems producing multiple products.

The framework here presented is an extension of the one introduced in (Bispo, 1997), which will now contemplate non-acyclic layouts, as well as a set of production bounds capable of inducing idleness in the production policies. These bounds will directly impact equation (6) below. For a more detailed description of the framework, we refer the reader to (Santos, 2016), of which this paper constitutes a synthesis.

### 2.1 Framework

Consider an eventually non-acyclic, multi-stage, multi-product, capacitated production system facing random

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demand, being a capacitated production system, a system whose machines have a limited amount of capacity available at any point of time. Each one of the $P$ products follows a specific routing pattern defined by a set $O^p$ of operations. Every set of operations $O^p$ is indexed from $K$ to 1 being $K$ the first operation (most upstream) and 1 the last. When analysing a given operation, the set $O^p$ will always have the information of which are its upstream and downstream operations. This information is what makes possible the generalization of the model to non-acyclic layouts.

### 2.2 Basic recursions

*Inventory Dynamic Equation* For the sake of simplicity, in the following set of equations, $I_n^{kp}$ and $P_n^{kp}$ refer, respectively, to the inventory and production values of operation $O^p(k)$ at the decision period n respectively. $d_n^p$ represents the demand value for product $p$ at the decision period $n$.

$$I_{n+1}^{kp} = \begin{cases} I_n^{1p} - d_n^p + P_n^{1p} & k = 1, \\ I_n^{kp} - P_n^{(k)^- p} + P_n^{kp} & \text{otherwise.} \end{cases} \quad (1)$$

Equation (1) describes the evolution of the inventory levels throughout the operation. The inventory level for the most downstream operation ($I^{1p}$), decreases by the the amount of product $p$ which leaves the system by means of the demand process and increases by the amount produced at this stage. The other production levels will see their inventory levels decrease by the amount of product that their downstream operations consume.

Note that $P_n^{(k)^- p}$ represents the amount of product $p$ produced at decision period $n$ for operation $O^p(k-1)$.

*Echelon Inventory Dynamic Equation* The echelon inventory of $O^p(k)$ at decision period $n$ is defined as:

$$E_n^{kp} = \sum_{x=1}^{k} I_n^{xp}. \quad (2)$$

Locally, a stage has usually no visibility over the downstream inventory levels and, for that matter, equation (2) is not applicable. The dynamic evolution of the echelon inventory may be described by means of an alternative equation:

$$E_{n+1}^{kp} = E_n^{kp} - d_n^p + P_n^{kp}. \quad (3)$$

Analysing the previous equation, the echelon inventory will grow with the amount of production at that corresponding stage and will decrease with the amount of products leaving the system.

*Shortfall Dynamic Equation* The shortfall is defined as the difference between the echelon base stock and the echelon inventory:

$$Y_n^{kp} = z^{kp} - E_n^{kp}. \quad (4)$$

It is possible to write a dynamic equation for the shortfall similar to the one for echelon inventory given by:

$$Y_{n+1}^{kp} = Y_n^{kp} + d_n^p - P_n^{kp}. \quad (5)$$

*Production net needs* Production net needs represent the production quantities that the system needs when there are no capacity bounds. In order to ensure that the capacity of the machines is not exceeded, production rules will be applied to the production needs in a posterior step. The production net needs may be bounded by upstream inventory or by the forced bounds imposed to the system. Let us define $\bar{I}^{kp}$ the production bound imposed to product $p$ at operation $k$. When there are no bounds, the system will always try to produce the sufficient amount to take the shortfall of the buffers to zero. The production net needs for a given product and operation are defined by:

$$f_n^{kp} = \begin{cases} \min\left\{(Y_n^{Kp} + d_n^p)^+, \bar{I}^{Kp}\right\} & k = K, \\ \min\left\{(Y_n^{kp} + d_n^p)^+, I_n^{(k)^+ p}, \bar{I}^{kp}\right\} & \text{otherwise.} \end{cases} \quad (6)$$

where $(x)^+ = \max\{0, x\}$. Note that the first machine of a production line will never be limited by inventory, since raw material is assumed to be always available.

The echelon base stock variables must respect the following rule $z^{kp} \geq z^{(k)^- p}$. As discussed in Bispo (1997), instead of continuously compare two consecutive multi-echelon base stock levels throughout the optimization procedure, it is preferable to use an alternative set of variables where this rule is simplified.

$$\Delta^{kp} = \begin{cases} z^{kp} & k = 1 \\ z^{kp} - z^{(k)^- p} & \text{otherwise.} \end{cases}$$

During the optimization procedure it is much easier to enforce that every $\Delta^{kp} \geq 0$ instead of making sure that the base stock variables are always non increasingly ordered.

*Priority Rule* Assuming that, every machine $m = 1, ..., M$ has a set of operations $J^m$ and that $J^m(x)$ , for $x = 1, ..., X^m$, is the operation that comes in the $x$th position on the priority list. That is, $J^m(1)$ is the operation with the highest priority and $J^m(X)$ has the lowest priority. The production decision for machine $m$ will be:

$$\begin{aligned} P_n^{(J^m(1))} &= \min\left\{ f_n^{(J^m(1))}, \frac{C^m}{\tau^{(J^m(1))}} \right\}, \\ &\vdots \\ P_n^{(J^m(x))} &= \min\left\{ f_n^{(J^m(x))}, \frac{C^m - \sum_{j=1}^{x-1} \tau^{(J^m(j))} P_n^{(J^m(j))}}{\tau^{(J^m(x))}} \right\}. \end{aligned} \quad (7)$$

Note: $\tau^{(J^m(x))}$ is the capacity required to produce one product unit on operation $J^m(x)$ in the machine $m$.

*Linear Scaling Rule* Product decision rule for operation $x$ of machine $m$:

$$P_n^{J^m(x)} = f_n^{J^m(x)} g_n^m, \quad (8)$$

$$g_n^m = \min\left\{ \frac{C^m}{\sum_{j=0}^{X} \tau^{J^m(j)} f_n^{J^m(j)}}, 1 \right\}, \quad (9)$$

*Operational Cost* The operational cost refers to the cost of stocking inventory and being penalized by backlogs. Let a single stage cost be defined as