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Dynamics analysis and time-optimal motion planning for unmanned quadrotor transportation systems☆,☆☆

Xiao Liang[a,b], Yongchun Fang*[,a,b], Ning Sun[a,b], He Lin[a,b]

[a] Institute of Robotics and Automatic Information System, Nankai University, Tianjin 300350, China
[b] Tianjin Key Laboratory of Intelligent Robotics, Tianjin 300350, China

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ABSTRACT

Aerial transportation is enjoying more and more popularity among disaster-oriented tasks, such as fire fighting, delivery of relief supplies, life-saving aid, and so on, which are infeasible for ground robots especially in complex terrains. However, the contradictory between transportation efficiency and payload swing suppression is an annoying problem, which badly limits the application of aerial transportation. In this paper, a time-optimal motion planning (TOMP) scheme is proposed with effective payload antiswing performance. To the best of our knowledge, it is the first minimum-time trajectory planning method designed for unmanned quadrotor transportation systems. Compared with existing methods, the proposed approach presents significant superiority in the sense that both nonlinear dynamics of the system and various constraints are taken into full consideration simultaneously. Specifically, the nonlinear system model is established using Lagrangian mechanics, based on which the augmented system is transformed into a nonlinear affine system regarding the acceleration/jerk as the control input without the need of the linearized system model. After discretization and approximation process, the time-optimal motion planning problem is converted into a standard nonlinear programming problem, wherein various practical constraints are considered, including the bounds of the payload swing, the quadrotor velocity, acceleration, and even jerk. Finally, the nonlinear programming problem is solved by the sequential quadratic programming (SQP) method. Experimental results are exhibited to illustrate the effectiveness and feasibility of the proposed approach.

1. Introduction

In the past decades, unmanned quadrotors are fast-growing in robotics field due to their excellent hovering capability, vertical taking off and landing (VTOL) ability, small size, satisfactory speed, noiseless operation, and easy maintenance [1–5]. Therefore, control of unmanned quadrotors has been widely studied, especially on the hovering problem [6], formation control [7–9], position tracking, and attitude stabilization [10–12], etc. These studies further extend the application fields of quadrotors, of which cargo transportation is a very attractive one receiving much attention recently.

In this paper, we focus on the transportation method with a cable-suspended payload beneath the quadrotor, which can also be extended to helicopters because of their similar dynamics. In this way, it is possible to transport toxic substances or objects of huge volume. Compared with the grasping method [13–15] which attaches the gripper to the quadrotor, the cable-suspended way requires less energy consumption because no extra mechanism or actuation are needed. In addition, the grasping method reduces the agility of the vehicle due to the increased rotational inertia. By contrast, object manipulation through cable suspension retains the agility of the quadrotor itself.

Similar to various transportation systems [16–19], the unmanned quadrotor transportation system needs to have the quadrotor accurately reach the desired position and the payload swing motion be suppressed during the process. This is an extremely challenging work as reflected by two aspects: (i) with underactuated characteristic, strong nonlinearities, and coupling [20–22] derived from the quadrotor itself, the control difficulty for the overall system is heavily increased; (ii) the payload swing motion, which is caused by the movement of the quadrotor, cannot be controlled directly. Thus, classical nonlinear methods for fully actuated systems are not applicable to the unmanned quadrotor transportation system.

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* Corresponding author.
E-mail addresses: liangx@mail.nankai.edu.cn (X. Liang), fangyc@nankai.edu.cn (Y. Fang), sunn@nankai.edu.cn (N. Sun).

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To achieve the desired objectives, some motion planning methods are proposed. In [23,24], the dynamic programming approach is utilized to generate a swing-free trajectory for the quadrotor. The proposed technique enables a swing-free flight by working backwards with a recursive relationship called Bellman equation. However, it completely relies on the discrete-time piecewise linearized model. In [25,26], the quadrotor-load system is proven to be a differentially-flat hybrid system with the load position and the quadrotor yaw being the flat outputs. According to the differential-flatness property, trajectories can be designed in the flat space by parameterizing the flat outputs as a series of basis functions on time. The coefficients of the basis polynomials are obtained by solving an optimization problem. This method is also applicable to address potential large dynamic swings of the payload. Alternatively, some reinforcement learning algorithms are also studied for quadrotor transportation systems. A finite-sampling and batch reinforcement learning algorithm is used to generate trajectories with minimal residual oscillations in [27]. With the designed problem-specific feature vector for value function approximation, this approach produces a swing-free trajectory for the desired state regardless of the starting position. In [28], a model-free least-square policy iteration (LSPI) method is proposed, which helps the quadrotor learn its own trajectory so as to make the suspended load track the reference trajectory.

Nevertheless, most existing methods ignore physical constraints and working efficiency requirements, which are two critical indexes in practical applications. In fact, the minimum-time control [29,30] for the unmanned quadrotor transportation system is of significant importance. Additionally, conventional acceleration-driven mode usually causes harmful vibration of motors due to discontinuous acceleration input. To effectively achieve quadrotor positioning and payload swing suppression in consideration of the aforementioned problems, in this paper, the complete process of dynamics analysis and motion planning are discussed, based on which a minimum-time control method is proposed. Specifically, to reduce the computation cost by Newtonian mechanics [23,24], the Lagrangian modeling approach is adopted. The system dynamics are divided into the inner part on the quadrotor rotational motion, and the outer one on the quadrotor translational motion and the payload swing motion. Compared with the work in [25,26], which describes the payload swing motion on S^2 by a unit vector, the intuitive payload swing representation adopted in this paper is more suitable and convenient for motion planning problems. Subsequently, the system model is rewritten into a nonlinear affine form with acceleration (alternatively jerk) being the control input, based on which a time-optimal motion planning problem is formulated by taking into account of various constraints for the state and control variables, including the upper and lower bounds of the swing angles, as well as the allowable quadrotor velocity/acceleration/jerk. To handle the aforementioned motion planning problem with dynamic constraints, Gauss pseudospectral method is then adopted to yield minimum-time planning for the system. After discretization and approximation of the state trajectory and the input vector by Lagrange interpolating polynomials, the original time-optimal problem is successfully transformed into an algebraic nonlinear programming problem, for which many existing methods are applicable to provide an effective solution. Hereto, a minimum-time trajectory under subjects to both state and control constraints for the unmanned quadrotor transportation system is constructed, whose superior performance is validated by numerous experimental results. The proposed planning scheme presents the following four main advantages compared with previous methods: i) various constraints of state and control variables can be considered simultaneously; ii) without the need of the linearized system model, the method is still reliable even in the presence of complex nonlinear dynamics; iii) time-optimal solutions can be obtained even under the above mentioned constraints; iv) it is worth noting that, to ensure that the quadrotor acceleration is continuous so as to avoid undesired vibration of the actuator, the construction of the jerk-driven mode is also provided.

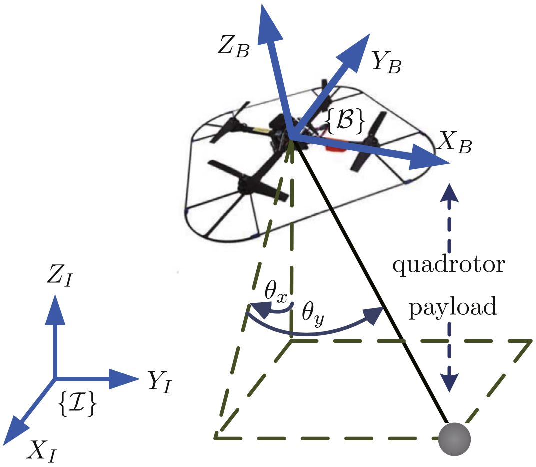

Fig. 1. Schematic for the unmanned quadrotor transportation system.

The rest of this paper is organized as follows. In Section 2, we present dynamics analysis on the overall system. Section 3 describes system transformation and the time-optimal trajectory planning procedure, in which both the acceleration-driven mode and the extended jerk-driven one are presented. Both numerical simulation and hardware experimental results are given to validate the effectiveness of the proposed planning scheme in Section 4. Finally, Section 5 provides some summaries of this paper.

2. Dynamics analysis

The studied unmanned quadrotor transportation system is depicted by Fig. 1, where the payload is suspended under the quadrotor's center of gravity. $\mathscr{I}$ represents a right hand inertia frame, $\mathscr{B}$ is the body-fixed frame. $\boldsymbol{\xi}(t) = [x(t),\ y(t),\ z(t)]^\top \in \mathbb{R}^3$ denotes the Euclidean position of the quadrotor with respect to frame $\mathscr{I}$, $\theta_x(t) \in \mathbb{R}$ and $\theta_y(t) \in \mathbb{R}$ are the payload swing angles; $R(t) \in SO(3)$ denotes the rotation matrix of the quadrotor from frame $\mathscr{B}$ to frame $\mathscr{I}$, $\boldsymbol{\Omega}(t) \in \mathbb{R}^3$ stands for the angular velocity of the quadrotor described in $\mathscr{B}$; M is the quadrotor mass, m is the payload mass, l denotes the rope length, $g \in \mathbb{R}$ is the gravitational acceleration, $J \in \mathbb{R}^{3\times3}$ represents the moment of inertia for the quadrotor with respect to frame $\mathscr{B}$; $f(t) \in \mathbb{R}$ is the applied thrust and $\tau(t) \in \mathbb{R}^3$ denotes the moment vector of the quadrotor in frame $\mathscr{B}$.

As widely done in the related literature on suspended systems [23,31–34], the following assumptions on transportation systems are made:

Assumption 1. The rope is inelastic and massless.

Assumption 2. The effects of air viscosity are negligible.

The modeling process is of critical importance for motion planning and system control. An intuitive expression is not only helpful for systematic understanding, but also a boost for the formulation of the trajectory planning problem. The unmanned quadrotor transportation system possesses eight degrees of freedom and presents strong coupling between the quadrotor translational motion and rotational motion [21,22,35,36], thus, it is difficult to model the system by utilizing Newtonian mechanics. In this paper, Lagrangian mechanics is performed. According to Fig. 1, by some geometric operations, we can first obtain the payload position $\boldsymbol{\xi}_p = [x_p,\ y_p,\ z_p]^\top$ as follows:

$$x_p = x + l\mathrm{S}_x\mathrm{C}_y, \tag{1}$$

$$y_p = y + l\mathrm{S}_y, \tag{2}$$

$$z_p = z - l\mathrm{C}_x\mathrm{C}_y, \tag{3}$$