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A new recurrent neural network with noise-tolerance and finite-time convergence for dynamic quadratic minimization[☆]

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ABSTRACT

To solve dynamic quadratic minimization, a nonlinearly activated integration design formula is first proposed in this paper with additive noises considered. Then, on the basis of such a design formula, a new recurrent neural network (RNN) is established to solve the dynamic quadratic minimization. Compared with the conventional Zhang neural network (ZNN) for this problem, the proposed RNN model possesses the outstanding finite-time convergence and the inherently noise-tolerant performance, and is thus called the versatile RNN (VRNN) model. In addition, the global stability, the finite-time convergence and the denoising ability of the VRNN model are proved by rigorous mathematical results in theory. The upper bound of the finite convergence time for the VRNN model is also analytically derived. Numerical simulative results are presented to validate the efficacy of the VRNN model, as well as its superior performance to the conventional ZNN model for dynamic quadratic minimization in the presence of various additive noises.

1. Introduction

Quadratic minimization, as a special case of nonlinear optimization, was widely encountered in diverse fields such as robotics, image processing, economics [1–3]. In [1], Nikolova and Chan presented an effective way to restore images by minimizing a quadratic cost function. In [2,3], kinematical control of robots were usually formulated as an optimal problem by minimizing a quadratic index with physics meaning. Therefore, the past decades made much progress in design and analysis of various approaches for quadratic minimization [4–7]. In [4], Xu presented a numerical iterative method to produce a sequence, which can converge to the unique solution of the quadratic minimization problem. In [5], Siebert and Veeser proposed an adaptive algorithm to solve quadratic minimization problem based on finite element functions, and the convergence of this algorithm to the exact minima was proved. In [6], Ruozzi and Tatikonda studied the behavior of reweighted message-passing algorithms for the quadratic minimization problem. In [7], Ye and Zhang presented polynomial-time procedures for solving quadratic minimization and proved the existence of a trajectory converging to an optimal solution. However, as pointed out in [8], numerical iterative algorithms have the serial-processing property, which may limit their computing power for large-scale real time application.

Because of excellent computation property, recurrent neural networks (RNNs) have become a growing concern in the past decade, with a lot of achievements gained by researchers [9–14]. For example, various RNN models have been presented and applied to model predictive control [15], the network identification [16], tracking control of nonlinear systems [17] and so on. Gradient neural networks (GNNs), as a classical RNN, were commonly used to solve various static problems effectively based on the core concept of the gradient descent [18–21]. However, it was pointed out that, when applied to solving dynamic (or, time-varying) problems, a lagging error was usually produced [20,21]. This phenomenon would affect the solution accuracy and limit the role of GNNs for real-time applications. In order to overcome the shortcomings of GNNs, a special kind of RNNs, called Zhang neural networks (ZNNs), were proposed and investigated for solving diverse dynamic problems effectively [22–26]. As compared to GNNs, ZNNs

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have a better convergence property for static problems solving, and the desired solutions of dynamic problems can be gained by adopting the time derivative information of varying coefficients [21,25]. In addition, to improve the convergence speed of ZNNs, a kind of finite-time ZNN (FTZNNs) were further proposed and applied to many engineering problems solving [27–31]. Compared with ZNNs and GNNs, FTZNNs had a best convergence property [27]. However, FTZNNs did not consider the impact of noises in the previous work. That is to say, FTZNNs were simulated in the ideal circumstances. In fact, noises are now everywhere in the real world, especially existing in the implementation of RNNs. Note that noises have a negative impact on the solution accuracy of RNNs. For better implementation of RNNs, the robustness to noises (i.e, the noise-tolerant ability) has to be considered in design process of RNNs.

Therefore, a new recurrent neural network is proposed in this paper for solving dynamic quadratic minimization in the presence of additive noises. Compared with ZNNs, the proposed RNN model not only possesses the finite-time convergence property but also has the inherently noise-tolerant ability, and is thus called the versatile RNN (VRNN) model. The core content of the VRNN model is based on a nonlinearly activated integration design formula that is quite different from ZNNs' formula. In addition, theoretical results about the global stability, the finite-time convergence and the denoising ability of the VRNN model are analyzed in details. Numerical simulative results validate the theoretical analyses of the VRNN model, as well as its superiority to the ZNN model for dynamic quadratic minimization in the presence of additive noises. Before ending this part, it is worth highlighting the novelty and contribution of this paper as follows.

(1) A nonlinearly activated integration design formula is proposed for modifying the existing ZNNs' design formula that cannot converge in finite time and inherently noise-tolerant. The current design formula fills this gap in a unified framework.
(2) A versatile recurrent neural network (VRNN) is proposed and analyzed in details for solving dynamic quadratic minimization problem in the presence of additive noises. To the authors' knowledge, this is the first RNN model to address this problem, simultaneously possessing the anti-noise ability and the finite-time convergence property.
(3) The global stability, the finite-time convergence, and the denoising ability of the VRNN model are proved in theory, which makes great progress, as compared with the previous theorems related to ZNN.
(4) Numerical comparative results are presented to verify the superiority of the VRNN model for solving dynamic quadratic minimization in the presence of the additive noises.

2. Problem description and ZNN

In this section, we are concerned with the following dynamic quadratic minimization equation [18,19,22]:

$$\min \quad x^{\mathrm{T}}(t)D(t)x(t)/2+w^{\mathrm{T}}(t)x(t), \tag{1}$$

where $x(t)\in R^n$ represents an unknown dynamic vector to be solved [with $x^{\mathrm{T}}(t)$ denoting the transpose of $x(t)$]. In Eq. (1), $D(t)\in R^{n\times n}$ and $w(t)\in R^n$ are known. The current work aims at finding the optimal solution of the above dynamic quadratic minimization within finite time in presence of additive noises. For ensuring the existence of the unique optimal solution for Eq. (1), it is assumed through the whole paper that $D(t)\in R^{n\times n}$ is a positive definite matrix at each time instant t [19,22]. Generally, in order to solve the above dynamic quadratic minimization, the Lagrangian multiplier method is first adopted to transform (1) into an equivalent dynamic system of linear equation [18,19,22]. Accordingly, we can define a Lagrangian function $f(t,x(t)):=x^{\mathrm{T}}(t)D(t)x(t)/2+w^{\mathrm{T}}(t)x(t)$. The partial derivative of $f(t, x(t))$ respect to $x(t)$ is thus derived below:

$$\nabla f(t,x(t))=\frac{\partial f(t,x(t))}{\partial x}=D(t)x(t)+w(t). \tag{2}$$

In addition, it can be easily concluded that the optimal solution of the dynamic quadratic minimization problem (1) can be gained by zeroing the partial derivative $\nabla f(t, x(t))$ at each time instant t. Thus, dynamic quadratic minimization (1) is equivalently reformulated as

$$D(t)x(t)+w(t)=0. \tag{3}$$

Then, according to Zhang et al.'s design method [22,25], by defining the error function $E(t)=\nabla f(t,x(t))=D(t)x(t)+w(t)$ and the ZNN design formula $\dot{E}(t)=-\gamma E(t)$, the original ZNN model for solving dynamic quadratic minimization problem (1) and the equivalent dynamic linear equation system (3) is derived as below:

$$D(t)\dot{x}(t)=-\gamma(D(t)x(t)+w(t))-\dot{D}(t)x(t)-\dot{w}(t), \tag{4}$$

where $\gamma>0$ is a scaling factor. For comparative purpose, the noise-polluted ZNN model for dynamic quadratic minimization problem (1) is directly extended as

$$D(t)\dot{x}(t)=-\gamma(D(t)x(t)+w(t))-\dot{D}(t)x(t)-\dot{w}(t)+\delta, \tag{5}$$

where δ denotes an unknown additive noise. It has been proved that the exponential convergence of the model (4) can be achieved for achieving the optimal solution of dynamic quadratic minimization in the free-noise situation [19,22]. However, as discussed before, the original ZNN model (4) cannot achieve the finite-time convergence, and does not consider additive noises. Thus, the solution accuracy of the original ZNN model (4) will be questioned in the presence of unknown additive noises. Therefore, as an extension to the previous work, we continue to study the impact of unknown additive noises to the original ZNN model (4) by analyzing the noise-polluted ZNN model (5).

3. Versatile recurrent neural network

In this section, based on the equivalent derivation from dynamic quadratic minimization to dynamic system of linear equations presented in the above, a nonlinearly activated integration design formula is first proposed in this section with additive noises considered. Then, the versatile recurrent neural network (VRNN) is established for the optimal solution of dynamic quadratic minimization. The detailed design process of the VRNN model is presented as follows.

In the first place, by following Zhang *et al.*'s design method [22,25], the error function based on the formulation of dynamic linear equation system (3) is chosen as below:

$$E(t)=D(t)x(t)+w(t)\in R^n. \tag{6}$$

Then, differing from the design formula related to ZNNs, a nonlinearly activated integration design formula for $E(t)$ is proposed as below:

$$\dot{E}(t)=-\gamma_1\mathcal{H}_1(E(t))-\gamma_2\mathcal{H}_2\left(E(t)+\gamma_1\int_0^t\mathcal{H}_1(E(\tau))\mathrm{d}\tau\right), \tag{7}$$

where argument t stands for time; $\gamma_1>0$ and $\gamma_2>0$ stand for the scaling factors of $\lim_{t\to\infty}E(t)=0$; and $\mathcal{H}_1(\cdot)$ and $\mathcal{H}_2(\cdot)$ stand for two monotone increasing odd activation function arrays, which can be the same or different.