ARTICLE IN PRESS

Discrete Optimization ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Discrete Optimization

www.elsevier.com/locate/disopt

Bi-criteria Pareto-scheduling on a single machine with due indices and precedence constraints

Yuan Gao, Jinjiang Yuan*

School of Mathematics and Statistics, Zhengzhou University, Zhengzhou, Henan 450001, People's Republic of China

ARTICLE INFO

Article history:
Received 11 March 2016
Received in revised form 21 February 2017
Accepted 23 February 2017
Available online xxxx

Keywords:
Bi-criteria
Pareto-scheduling
Positional due indices
Precedence constraints

ABSTRACT

We consider the Pareto-scheduling with bi-criteria on a single machine in which each task has a positional due index. Two bi-criteria problems are considered: (a) Pareto-scheduling with two agents A and B for minimizing the total completion time of A-tasks and a maximum cost of B-tasks with precedence constraints. (b) Pareto-scheduling under precedence constraints for minimizing two maximum costs of tasks. We show in this paper that the two problems are both solvable in polynomial time. The second result also implies that the Pareto-scheduling under precedence constraints for minimizing two agents' maximum costs is solvable in polynomial time.

1. Introduction

Suppose there are n tasks $\mathcal{T} = \{T_1, T_2, \ldots, T_n\}$ to be scheduled on a common machine without preemption. In the following, we introduce the *Pareto-scheduling problem* for minimizing two objective functions γ' and γ''.

A feasible schedule ψ of the n tasks in a given machine environment is called a *Pareto schedule* if there exists no other feasible schedule φ such that

$$(\gamma'(\varphi), \gamma''(\varphi)) \leq (\gamma'(\psi), \gamma''(\psi)) \quad \text{and} \quad (\gamma'(\varphi), \gamma''(\varphi)) \neq (\gamma'(\psi), \gamma''(\psi)).$$

In this case, $(\gamma'(\psi), \gamma''(\psi))$ is also called a *Pareto point*. The aim of the problem is to find all Pareto points and, for each Pareto point, a corresponding Pareto schedule. Following the notation introduced in T'kindt and Billaut [1], the Pareto-scheduling problem can be denoted by $\alpha|\beta|^{\#}(\gamma', \gamma'')$, where α represents the machine environment and β represents the job characteristics or the feasibility conditions. In this paper, our research is restricted on a single machine. Then we have $\alpha = 1$.

* Corresponding author.
E-mail address: yuanjj@zzu.edu.cn (J. Yuan).

http://dx.doi.org/10.1016/j.disopt.2017.02.004
1572-5286/

Pareto-scheduling has been extensively studied in the literature. The methodology and development in the research of Pareto-scheduling can be found in Hoogeveen [2], T'kindt and Billaut [1], Perez-Gonzalez and Framinan [3], and Agnetis et al. [4].

In the two-agent scheduling (which was first introduced in Agnetis et al. [5]), there are two agents A and B and the n tasks $\mathcal{T} = \{T_1, T_2, \ldots, T_n\}$ are partitioned into two parts (subsets) $\mathcal{T}^A = \{T_1, T_2, \ldots, T_{n_A}\}$ (called A-tasks) and $\mathcal{T}^B = \{T_{n_A+1}, T_{n_A+2}, \ldots, T_n\}$ (called B-tasks). The n tasks need to be scheduled on a single machine with each agent having his scheduling cost of his tasks. Among many results in the work of Agnetis et al. [5], they presented an $O(n_A^2 n_B \log n_A + n_A n_B^3)$-time algorithm for problem $1 \parallel {}^{\#}(\sum C_j^A, f_{\max}^B)$. After their pioneering work, two-agent scheduling had been extensively studied in the literature. A detail review for the research on two-agent scheduling can be found in Agnetis et al. [4].

For problem $1 \parallel {}^{\#}(f_{\max}, g_{\max})$, Hoogeveen [6] presented an $O(n^4)$-time algorithm. For problem $1 \parallel {}^{\#}(f_{\max}^A, g_{\max}^B)$, Agnetis et al. [5] presented an $O(n_A^3 n_B + n_A n_B^3)$-time algorithm. However, as we will state in Section 4 of this paper, there exists a flaw in the deduction for the related result in [6], and the deduction for the related result in [5] is confusing.

Following the work of Agnetis et al. [5] for problems $1 \parallel {}^{\#}(\sum C_j^A, f_{\max}^B)$ and $1 \parallel {}^{\#}(f_{\max}^A, g_{\max}^B)$ and Hoogeveen [6] for problem $1 \parallel {}^{\#}(f_{\max}, g_{\max})$, we study in this paper the Pareto-scheduling on a single machine with positional due indices and precedence constraints. In the problem, we have n tasks $T_1, T_2, \ldots, T_n$ and each task T_j is associated with a position index $k_j \in \{1, 2, \ldots, n\}$, called the *positional due index* (due index) of task T_j. In a feasible schedule, each task T_j is required to be scheduled in a position with index at most k_j. Given a schedule ψ of the n tasks $T_1, T_2, \ldots, T_n$, we say that the *position index* of a task T_j in schedule ψ, denoted by $\psi[T_j]$, is x for some $x \in \{1, 2, \ldots, n\}$ if task T_j is scheduled in the xth position in schedule ψ. Then the schedule ψ is feasible if it satisfies the *due-index constraints* $\psi[T_j] \le k_j$ for all $j = 1, 2, \ldots, n$. In a certain extent, the due indices of tasks are similar to the deadlines of tasks. The difference is that the due indices restrict the positions of tasks and the deadlines restrict the completion times of tasks.

For the Pareto-scheduling problem to minimize the total completion time $\sum C_j$ and the maximum cost $f_{\max}$ on a single machine with due-index constraints, Gao and Yuan [7] presented an $O(n^4)$-time optimal algorithm which is developed from an $O(n^3 \log \sum p_j)$-time algorithm for the weakened version of the same problem without due-index constraints presented in Gao and Yuan [8].

In this paper, we mainly study two Pareto-scheduling problems. The first problem is the two-agent Pareto-scheduling on a common machine with due-index constraints for all tasks and precedence constraints only for the B-tasks for minimizing the total completion time of A-tasks $\sum C_j^A$ and the maximum cost of B-tasks $f_{\max}^B$. Then the problem can be denoted by

$$1|\psi[T_j] \le k_j, \text{prec}^B|^{\#}\left(\sum C_j^A, f_{\max}^B\right),$$

where "$\psi[T_j] \le k_j$" denotes the due-index constraints and "prec^B" denotes the precedence constraints of the B-tasks (that is, $T_i \prec T_j$ means that task T_i must be processed before task T_j in any feasible schedule).

The second problem is the Pareto-scheduling on a common machine with due-index constraints and precedence constraints for minimizing two maximum costs $f_{\max}$ and $g_{\max}$. Then the problem can be denoted by

$$1|\psi[T_j] \le k_j, \text{prec}|^{\#}(f_{\max}, g_{\max}),$$

where "prec" denotes the precedence constraints of all tasks. In the counterpart of two-agent scheduling, the problem is denoted by

$$1|\psi[T_j] \le k_j, \text{prec}|^{\#}(f_{\max}^A, g_{\max}^B),$$

where $f_{\max}^A$ and $g_{\max}^B$ are the max-form objective functions of agent A and agent B, respectively.

Please cite this article in press as: Y. Gao, J. Yuan, Bi-criteria Pareto-scheduling on a single machine with due indices and precedence constraints, Discrete Optimization (2017), http://dx.doi.org/10.1016/j.disopt.2017.02.004