Available online at www.sciencedirect.com
ScienceDirect
IFAC PapersOnLine 50-1 (2017) 5289–5294

Delay Variability Optimization Using Shockwave Theory at an Undersaturated Intersection

Reza Mohajerpoor * **Meead Saberi** * **Mohsen Ramezani** **

* *Institute of Transport Studies, Department of Civil Engineering, Monash University, Melbourne, Australia*
** *School of Civil Engineering, The University of Sydney, Australia*

Abstract: Signal optimization is one of the most crucial problems in the traffic flow theory. Delay at signalized intersections is the main component of travel time in urban transportation networks. This paper investigates an analytical approach based on the shockwave theory to estimate the delay of each vehicle joining the queue, and minimize the total delay and delay variability at an undersaturated intersection. The optimizations are carried out for an intersection with and without loss time and are formulated as convex programs. The global optimal cycle length and splits are attained to minimize the delay variability.

Keywords: Delay distribution; fundamental diagram; travel time reliability; convex optimization

1. INTRODUCTION

Signal optimization of isolated intersections has been studied for a long time (Wardrop, 1952; Webster, 1958; Gazis, 1964). The seminal work of Webster (1958) proposed an analytical formula for the optimal cycle length at an isolated undersaturated intersection. However, as demonstrated in Wagner et al. (2014), the formula is over-simplified and does not necessarily provide the optimal solution. Furthermore, the introduction of the *kinematic shockwave* theory (Lighthill and Whitham, 1955; Richards, 1956) provided alternative methods for modelling and analyzing the traffic flow of arterials and intersections (see e.g. Dion et al. (2004); Skabardonis and Geroliminis (2008); Liu et al. (2009); Cheng et al. (2011); Ramezani and Geroliminis (2015)). The delay estimation at intersections using the shockwave theory was first presented in Michalopoulos and Pisharody (1981), where the authors considered a variable density at the discharge phase of the intersection, which itself resulted in a rigorous formula. Michalopoulos et al. (1981) proposed a heuristic signal control algorithm for an over-saturated isolated intersection. Dion et al. (2004) presented a comprehensive literature review on the delay modelling approaches and classified them into deterministic queueing algorithms, shockwave based models, and microscopic simulation based approaches.

The literature on the optimization of traffic networks can be classified into studies that (i) focus on multiple interconnected intersections (Dinopoulou et al., 2006; Kosmatopoulos et al., 2007; Ramezani et al., 2016), (ii) investigate network-wide signal control e.g. (Diakaki et al., 2002; Geroliminis et al., 2013; Ramezani et al., 2015; Keyvan-Ekbatani et al., 2012), and (iii) optimize a single isolated intersection. Gazis (1964) propose an optimization approach based on the Pontryagin's minimum principle for an oversaturated isolated intersection, using a semi-graphical methodology and employing the queueing theory.

The *queueing theory* has been widely employed for the signal timing optimization, relying on various numerical optimization approaches (Haddad et al., 2010; Aboudolas et al., 2010; Ioslovich et al., 2011; Varaiya, 2013b,a; Haddad et al., 2014). Haddad et al. (2010) proposed a discrete-event max-plus approach to model the traffic flow of a two-way isolated undersaturated intersection and minimize a weighted sum of the red phases of the intersection as the optimization criterion. It assumes a *discrete* model and a *given fixed cycle-time* in the modelling and optimization stages. Later Haddad et al. (2014) extended the theory to consider the assumption of the lower and upper bounds of the green phase of each intersection. In addition, the above theory is applied in Ioslovich et al. (2011) to optimize an oversaturated intersection assuming a continuous model as in Gazis (1964). Moreover, Varaiya (2013b) proposed the concept of max-pressure and apply it on the *store and forward* queueing model to stabilize an arbitrary network of correlated intersections under uncertain demands and turning ratios.

Nevertheless, studies based on the store and forward queueing model do not provide full spatial and temporal characteristics of queuing dynamics. Specifically the spill-back phenomenon cannot be fully captured. In addition, it is well-known that when the queue in an approach exceeds the detector's location, models based on the queuing theory face observability problems in practical cases (Liu et al., 2009; Cheng et al., 2011).

Accordingly, this paper investigates the traffic signal control problem at intersections using the shockwave theory. An analytical model is presented for estimating the delay of a vehicle joining the queue, and the distribution of

2405-8963 © 2017, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.622

intersection delay. Using the model, the paper formulates the optimal cycle time and the green phase allocations at an *undersaturated* intersection considering constant flow rates per cycle in two cases of with and without nominal loss times (LTs). The optimization of the signals is carried out by minimizing the following objective functions: (i) total delay and (ii) delay variability. The delay distribution at the intersection is established, and the variance of delay is introduced as a criterion for increasing the performance reliability of the signal control. Undersaturation and spill-back avoidance are formulated to define the optimization constraints.

NOMENCLATURE

q_i^{a}	The arrival flow at Approach i; [veh/unit time]
q_i^{c}	The saturation flow (or capacity) at Approach i; [veh/unit time]
k_i^{a}	The arrival traffic density at Approach i; [veh/unit distance]
k_i^{c}	The saturation traffic density at Approach i; [veh/unit distance]
k_i^{j}	The jam density at Approach i; [veh/unit distance]
R_i, G_i	Respectively, the red phase and green phase of Approach i in the cycle; [unit time]
C	The cycle time; [unit time]
L_i, L	The loss time at Approach i, and the total loss time, respectively; [unit time]
t_i	The time of joining the queue at Approach i; [unit time]
$x_i(t_i)$	The position of the back of the queue in Approach i as a function of t_i; [unit length]
$D_i(t_i)$	Delay of the vehicle at Approach i joining the queue at t_i; [unit time]
x_i^{j} and t_i^{j}	The position and the time of queue clearance at Approach i, respectively
$\hat{i} \in \Omega$	The complement of i in the set Ω

2. PRELIMINARIES OF DELAY ESTIMATION

In this section the delay of every vehicle joining the queue at an intersection without LT is estimated using the shockwave theory. Consequently, the effect of drivers' reaction times is also amended in the estimation. It is emphasized that the formulas developed in this section are aimed at helping with the optimization and control of traffic at the intersection. Delay formulas have been extensively studied previously using various methods.

Variable $t_i \in [0, t_i^{\mathrm{j}}]$, $i = \{1, 2\}$ is defined as the time of joining the queue for vehicles at Approach i. The methodology of the paper is based on the time-space diagram (TSD) that contains the trajectory of every vehicle entering and exiting an intersection, and the fundamental diagram (FD) that relates the traffic flow to the traffic density based on the characteristics of the road. TSD and FD of an approach are related through the kinematic shockwaves. A shockwave is a wave created due to any abrupt change in the states of traffic flow (Lighthill and Whitham, 1955). Fig. 1 depicts TSDs, FDs, and shockwaves of the two approaches at a two-phase undersaturated intersection with constant inflows.

To derive analytical formulation of delays we assume: (A1) the transition of every individual vehicle between the free flow speed to zero occurs with an *infinite acceleration*; (A2) the average arrival and discharge flows and speeds of each approach are constant and known; (A3) a triangular FD is considered (Fig. 1(b)); (A4) the intersection is two-phase; and (A5) the LT of each approach at the intersection is known and constant. Although only two phases are considered, this does not fail the generality of the paper, as it is well-understood that the multiple-phase problem is an extension of the two-phase model (Webster, 1958; Haddad et al., 2010; Varaiya, 2013a).

The shockwaves are bold solid lines depicted in Fig. 1 and the trajectories of vehicles are shown using thin directed lines. The slope of the shockwave from free flow to jam density conditions in Approach $i = \{1, 2\}$ is equal to the slope of J_iA_i in its FD. Analogously, the transition from jam state to the saturation flow can be described through a shockwave with the slope of the line J_iC_i in the FD. This relation is helpful in deriving the formulas of the paper.

In addition, calculating the points $(t_i^{\mathrm{j}}, x_i^{\mathrm{j}})$, $i = \{1, 2\}$, provides an estimation of the back of the queue, which is essential in analyzing the existence of any spillback for non-isolated intersections, and thus devising an appropriate signal setting to avoid spillback.

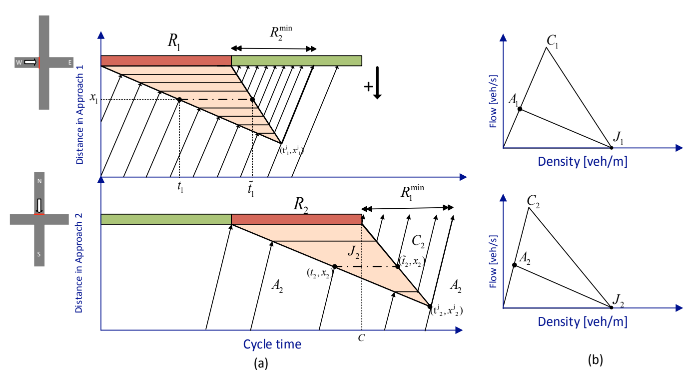

Fig. 1. (a) The time-space, and (b) the FDs of each approach at a two-phase isolated intersection.

Since the slopes of the arrival and discharge shockwaves can be obtained from the FD, the queue clearance point $(t_i^{\mathrm{j}}, x_i^{\mathrm{j}})$, $i = \{1, 2\}$, can be estimated as:

$$t_i^{\mathrm{j}} = \beta_i R_i, \tag{1}$$

$$x_i^{\mathrm{j}} = \frac{q_i^{\mathrm{a}}}{k_i^{\mathrm{j}} - k_i^{\mathrm{a}}} t_i^{\mathrm{j}}, \tag{2}$$

where

$$\beta_i \triangleq \frac{k_i^{\mathrm{c}}\left(k_i^{\mathrm{j}} - k_i^{\mathrm{a}}\right)}{k_i^{\mathrm{j}}\left(k_i^{\mathrm{c}} - k_i^{\mathrm{a}}\right)} > 1.$$

Since the vehicle that arrives to the queue at time t_i starts moving at time $\tilde{t}_i$ at the same location (see Fig. 1(a)), the delay of the vehicle is

$$D_i(t_i) = \tilde{t}_i - t_i \qquad \forall t_i \in [0, t_i^{\mathrm{j}}]. \tag{3}$$

Hence, the position of the back of the queue at time t_i is formulated as