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Constrained multi-objective optimization of storage ring lattices

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A B S T R A C T

The storage ring lattice optimization is a class of constrained multi-objective optimization problem, where in addition to low beam emittance, a large dynamic aperture for good injection efficiency and improved beam lifetime are also desirable. The convergence and computation times are of great concern for the optimization algorithms, as various objectives are to be optimized and a number of accelerator parameters to be varied over a large span with several constraints. In this paper, a study of storage ring lattice optimization using differential evolution is presented. The optimization results are compared with two most widely used optimization techniques in accelerators—genetic algorithm and particle swarm optimization. It is found that the differential evolution produces a better Pareto optimal front in reasonable computation time between two conflicting objectives-beam emittance and dispersion function in the straight section. The differential evolution was used, extensively, for the optimization of linear and nonlinear lattices of Indus-2 for exploring various operational modes within the magnet power supply capabilities.

1. Introduction

The lattice optimization for the low emittance synchrotron light sources is an involved task, which belongs to a class of multi-objective optimization problem. One needs to optimize various objectives simultaneously [1]. For a synchrotron light source, its brightness is the main figure of merit. It is inversely proportional to the product of horizontal and vertical beam emittances. The beam emittance [2] is a complicated function of the lattice properties driven by the distribution of dipole and quadrupole magnets and their strengths. Minimizing the beam emittance is not the only objective of the lattice design, instead one also needs to optimize for the operational feasibility, for accommodating larger number of insertion devices, for achieving good injection efficiency and sufficient beam lifetime etc. The sextupole magnets with high strengths are the integrated elements of the low emittance lattices to correct for high natural chromaticities, which may lead to shrinking of dynamic aperture (DA). Keeping in view that all types of magnets are to be used, low emittance and large DA are the ultimate optimization goals. In some machines, there is a requirement of getting special distribution of lattice functions, for example, low beta is required for minimizing the effects of insertion devices in operation and high horizontal beta is required for the conventional injection in storage rings. The smaller momentum compaction factor is desirable for producing the shorter bunches. The optimization of the distribution of the lattice functions and still minimizing the beam emittance and increasing DA are some of the conflicting requirements.

Earlier, various traditional approaches were applied for optimizing the lattice parameters of the storage rings. In these approaches, one needs to first find a stable solution, often by trial and error, that roughly meets the desired properties, and then goal is to locally improve around it. A technique called GLASS (GLobal scan of All Stable Settings) [3] was proposed in which the properties of the lattices are systematically calculated for all stable solutions. However, because the computing time is exponentially large, depending on the numbers of variables and step sizes, GLASS currently is unfeasible for problems with more than 4–5 variables [4]. Usually, the variables that govern such problems are related to one another in a very complicated way, and finding the best combination of them can be a challenging task. Normally, the relations between these decision variables can be translated into objective functions, and their values can be interpreted as a measure of the quality offered by that particular combination of the decision variables to a particular aspect of the solution. The n-dimensional optimization problems with m individual objectives can be stated as [5]:

$$Minimize/Maximize: \; F(\boldsymbol{x}) = \left[f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), f_3(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})\right]; \qquad (1)$$

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$x_i^{(L)} \le x_i \le x_i^{(U)}, \quad i = 1, 2, 3, \ldots, n$

where, $\boldsymbol{x}$ is a vector of n dimensional decision variables $\boldsymbol{x} = (x_1, x_2, x_3, \ldots, x_n)$. Here, $x_i^{(L)}$ and $x_i^{(U)}$ are the lower and upper bounds of the decision variables x_i. The functions $f_1(\boldsymbol{x})$ through $f_m(\boldsymbol{x})$ indicate the individual objective functions. In almost all practical cases, finding good solutions is not a problem, but finding the best solution is much more involved. Moreover, for most problems encountered in practice, the objective functions f_1 *(x)* through f_m *(x)* are highly nonlinear, non-differentiable, or have no way of determining initial estimates close to the global optimum. The practical approach to tackle such problems is to use the meta-heuristic optimizers, which use a population of trial solutions, and apply certain probabilistic rules to generate a new population, which converge to the global minimum of the objective functions with high probability. Over the years, many such algorithms have been developed, of which the Genetic Algorithm (GA) [1,5–10], Particle Swarm Optimization (PSO) [11–13] are most widely used in optimization of accelerator parameters.

GA is a class of evolutionary algorithms of which Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [5] is more popular, which is based on natural evolution of crossover, mutation and selection. The PSO is a class of swarm intelligence algorithm. PSO follows the self-organizing behaviour of social animal living in a group. It has been recently used to optimize the Linac operation and nonlinear dynamics of storage rings [11–13].

It has been demonstrated that both the algorithms—GA and PSO are powerful and effective in solving the problems for highly nonlinear objectives, which have a large number of local optima. Comparative studies of these two techniques have been performed in Ref. [13] for optimizing the momentum and dynamic apertures of SPEAR3 storage ring. Two cases of initial population generation were considered: (i) purely random distribution within lower and upper bound, and (ii) initial population seeded with the nominal good solution and a small random distribution around it. It was reported that PSO converges faster than GA, and is not dependent on the distribution of initial population, while GA depends on initial population distribution considerably. In order to check this further, a performance comparison of these two algorithms by applying them to a problem with two objective functions—beam emittance and dispersion function at the centre of straight section are performed for the case of Indus-2. It was found that if initial population are randomly generated, then PSO and GA give almost equivalent results, however a diversity in the variable space is adequate in case of PSO.

Some authors have applied another class of evolutionary algorithm, called Differential Evolution (DE) [14], which is based on the globalized pseudo-derivatives. DE is a very powerful and simple population based algorithm like genetic algorithms using the similar operations—crossover, mutation and selection. The main difference in constructing the better solutions is that the genetic algorithms rely on crossover, while DE relies on mutation operation. This main operation is based on the differences of randomly sampled pairs of solutions in the population. The algorithm uses mutation operation as a search mechanism and selection operation to direct the search towards the prospective regions in the search space. The DE algorithm also uses a non-uniform crossover that can take the child vector parameters from one parent more frequently than it does from others. DE is an additional recipe for the optimization algorithms and has received significant interest, in the field of optimization of the complicated accelerator parameters [15–17]. A brief introduction to DE algorithm is described below.

The DE algorithm uses the mutation operation based on scaled differences of the parent solutions to generate the next generation candidates or trial parameter vectors for optimization. Originally, Storn and Price proposed five different mutation processes or strategies in Ref. [14]. For the present study, we have selected a 'rand-to-best' strategy [16,17], which gives a balance between fast convergence and robustness of the algorithm. In this strategy, a trial parameter vector v_i at generation *j* is generated according to

$$v_{i,j} = x_{i,j} + F_1\left(x_{b,j} - x_{i,j}\right) + F_2(x_{r_1,j} - x_{r_2,j}) \tag{2}$$

where, r_1 and r_2 are integer indices chosen randomly from the interval *[1, NP]* and are different from the running index *i*, $x_{b,j}$ is the best solution vector among *NP* population members at the generation *j*. F_1 *is* a weight factor for the combination between best and current parent vector, and F_2 is a scaling factor, which controls the amplification of the differential vectors and is known as mutation-weighting factor. Too large values of F_1 and F_2 provide the diversity in the solution space and too small values results in fast convergence. Normally there values are in the range 0–2.

To further increase the diversity in solution space, a crossover operation between the target vector $x_{i,j}$ and mutant vector generate $v_{i,j}$ at generation *j*, is performed. This operation combines the two vectors into a new trial vector $u_{i,j}$ as per the following rule:

$$u_{i,j} = \begin{cases} x_{i,j}, & if\ rand\,(j) \ge CR \\ v_{i,j}, & otherwise \end{cases} \tag{3}$$

where *rand()* is a uniform random number between 0 and 1, and *CR* is the crossover probability. CR, F_1 and F_2 are three important controlling parameters of the algorithm. The newly generated trial solution $u_{i,j}$ is checked against the parent solution $x_{i,j}$ through the selection operations like in NSGA-II.

Very recently, various DE mutation strategies have been used to optimize the photoinjector beam dynamics [15,16], and DA of the future collider [17]. In these studies, the value of *CR* is fixed at 0.8, $F = F_1 = F_2$ and takes the random value between 0 and 1 at every generation. The advantage of these optimization methods is that they allow to find globally optimal solutions, when a large numbers of fit parameters are used, while showing the trade-offs in objective functions within the acceptable computing time.

In this paper, all three multi-objective optimization algorithms-GA, PSO and DE are used to compare their performance for the case of linear lattice optimization of Indus-2 storage ring. The Indus-2 lattice is described in Section 2. In Section 3, a study for the optimization of beam emittance and the dispersion function in the straight section is presented. This uses strengths of all five quadrupole families as the variables. The results obtained by GLASS scan are also presented. The outcome of the study is that DE produces best Pareto optimal front compared to other algorithms in almost equal computation time. The optimization to achieve low momentum compaction factor together with lower beam emittance are discussed in Section 4. In Section 5, we extend the study to explore the alternate low and high beta lattices for commissioning of insertion devices (IDs) and satisfying the off axis injection. In this mode, DE was used for simultaneous optimization of the beam emittance and DA, including two sextupole family strengths as additional two variables. Lastly, in Section 6, summary and future possibilities of applying the DE for upgrade of Indus-2 lattice are presented.

2. Indus-2 lattice

Indus-2 [18,19] is a 3rd generation synchrotron light source located at Raja Ramanna Centre for Advanced Technology (RRCAT) Indore, India, which is optimized for the generation of photons at a critical wavelength of 2 Å. The ring is ~172.5 m in circumference and consists of eight super-periods. The lattice structure of one super-period is a double bend achromat of the expanded Chasmann Green type. The lattice functions together with arrangement of magnetic elements in a super-period are shown in Fig. 1. At present, Indus-2 is being operated at relaxed optics with beam emittance of 135 [nm rad] at 2.5 GeV. Basic operational parameters of Indus-2 are given in Table 1. The Indus-2 is running in the round the clock mode for beamline experiments and all hardware are fixed all along the circumference of the ring. For tuning the machine performance by shaping the lattice parameters for various