Optimum design and damage control for load sequences

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A B S T R A C T
A probabilistic approach to decision-optimal design and damage control is developed for structural systems that can gradually accumulate damage by nonlinear behavior under sequences of dynamic loads, whose occurrence can be idealized by renewal stochastic processes. To minimize consequences and damages during the life cycle of a structure, a damage threshold is established as a measure of damage control. If structural damages are lesser than such a damage threshold the structure is not repaired, otherwise the structure is repaired or rebuilt. The proposed approach is generalized and capable of describing particular cases for the optimization of expected losses. One of them is the well-known case used as a basis for many current design criteria in which it is assumed that the structure is repaired or rebuilt systematically after some damage or failure. The present work extends the ideas and models reported in several seminal papers. However, the proposed approach has the advantage that it takes into account cumulative structural damage over time, allows evaluating objectively the cost of damages and sets an optimum repairing damage threshold. Finally, the probabilistic formulation is illustrated through its application to a building subjected to sequences of earthquakes.

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1. Introduction

It is now recognized that design codes should ethically generate optimal designs for society in the sense that they have to consider protecting people and property [1]. A useful tool to achieve this goal is decision theory [2,3]. On the basis of this theory and according to the annals of UNAM (National University of Mexico) in 1968, Esteva [4] proposed the first model of design decisions for structures subjected to cyclic loads (earthquakes) that occur sporadically according to a stochastic Poisson process. Later on, between 1971 and 1974, Rosenblueth and Mendoza [5], Rosenblueth [6] and Hasofer [7] generalized the ideas of [4], by considering that loads are generated according to a stochastic renewal process, in which the Poisson process is only a particular case. In these works, the optimization (minimizing the present value of expected losses or maximizing benefits) is carried out by taking into account only initial and failure costs, because the formulation is related exclusively to a failure model. These kinds of models consider the contributions to total losses of all failure events occurring over time, since the structure is rebuilt each time it fails. Then, in 1976, Rosenblueth [8] additionally considered explicitly the inclusion of the expected damage losses conditioned on the structural survival. Long after, in 2000, Rackwitz [9] took the Rosenblueth’s and Hasofer’s approach [5–8] and made some improvements to the model, but without considering the damage and the associated damage costs conditioned on the structural survival. However, in other works [10–12] damage and repair costs are taken into account in the optimization process. This kind of model, based on renewal process, has been used by some researchers to perform engineering applications in the optimization of structural systems [13–16].

A fundamental assumption in all these works indicates that “the structural capacity is immediately restored after each load event causing damage”, which implies that the failure probability for uncertain load events is constant and even those damages that are not identified or visible are repaired. This means that there is no cumulative damage and every time that the structure is exposed to the next load its capacity remains intact. This assumption is also the backbone of modern structural design codes, because the design criteria refer to the behavior of structural systems with intact mechanical properties subjected to dynamic loads with different probabilities of occurrence. Codes do not explicitly take into account changes in the vulnerability of the structure during its service life, let alone specify acceptable damage thresholds. The above assumption has the advantage that mathematical analysis leads to simple analytical models and results. However, in
engineering practice the repair or rebuild decisions are made from an assessment of structural integrity, and in many cases this assessment is virtual and subjective. Therefore, the assumption of systematically repairing or rebuilding the structure at any level of damage after each load event is difficult to satisfy, whereas in other cases, it may be appropriate.

On the other hand, under this kind of approach, neither the damage probabilities are estimated from an explicit formulation nor the damage costs are explicitly estimated for given levels of damage. For this reason, the damage costs are estimated roughly, typically associated with a load intensity indicator. Apparently, only the initial costs and expected consequences due to structural failures are described adequately, because the stochastic formulation [4–7,9] only considers the failure condition, meanwhile the probability of damages and damage costs related to the structural survival associated with each load are not explicitly considered.

This paper describes an explicit and generalized probabilistic formulation to make decisions for design and damage control of structural systems whose structural vulnerability can change during its planned service life due to the impact of stochastic loads with uncertain intensities. The development follows the ideas of the above mentioned seminal papers [4–9]. The first part of this work outlines the basic stochastic characteristics useful to describe the evolution of damage in structural systems. After this, a generalized formulation and a particular case are described to quantify the present value of the expected losses of structures with cumulative damage. Finally, the proposed formulation is applied to a reinforced concrete building subjected to seismic sequences.

2. Stochastic properties of cumulative damage

This section outlines the basic stochastic properties to develop the generalized optimization model presented in the next sections. In this work we focus only on the probability density functions of the time to the n-th, n = 1, 2, . . . , exceedance of a given damage threshold.

2.1. Probability functions of the time to the n-th exceedance of a given damage threshold

Cumulative damage over time t is described by the random function D(t); it takes values in the interval [0, 1]. D(t) = 0 indicates nonstructural damage and D(t) > 1 indicates total damage. D(t) is represented as the sum of random increments of damage ΔDn(t), as follows

\[ D(t) = \sum_{n=1}^{N(t)} \Delta D_n(t - S_n, Y_n) \]  (1)

\( N(t) \) describes the cumulative numbers of load events that have impacted the structure over time through a stochastic renewal process [17]. Damage increments ΔDn(t), n = 1, 2, . . . are random variables that quantify on a dimensionless uniform scale [0, 1] the potential structural degradation each time that the structure is subjected to a dynamic load. Sn denotes the random variable of the time to the occurrence of the n-th load with intensity \( Y_n = Y \), also treated as a random variable. Here \( f_Y(y) \) is the probability density function of the intensities of dynamic loads. The probability \( p[D(t) > d] \), that at time \( t \), \( D(t) \) exceeds a given value of \( d \), can be written as

\[ p[D(t) > d] = \sum_{n=1}^{N(t)} \left[ \sum_{i=t}^{d} \Delta D_i(t - S_i, Y_i) > d \right] \]  (2)

For conciseness, \( \Delta D_0 = \Delta D_0(t) \) and \( D_0 = \sum_{i=1}^{N(t)} \Delta D_i \). Furthermore, \( D_0 = \Delta D_0 \) if \( d < 0 \). Thus, the damage \( D_n \) at the n-th occurrence can be expressed as \( D_n = D_{n-1} + \Delta D_n \). According to this, \n
\[ p[D_n > d] = \int_0^d p[\Delta D_n > d - x] f_{\Delta D_n}(x) dx \]  (3)

Each increment is conditioned on the previous state of structural integrity. Here, \( f_{\Delta D_n}(\cdot) \) is the probability density function of damage state to the occurrence of the \((n - 1)\)-th dynamic load, which is obtained as the derivative with respect to \( d \) of \( p[D_{n-1} < d] \). Regarding that damage remains constant between load events and that the increment of damage \( \Delta D_n \) not only depends on the last state of damage but also on the load of intensity \( Y = y \), Eq. (3) can be expressed in extended form as

\[ p[D_n > d] = \int_0^d \int_0^d p[\Delta D_n(y) > d - x] f_Y(y) f_{\Delta D_n}(x) dx dy \]  (4)

The probability distribution function of cumulative damage \( F_{D_n}(d) = 1 - p[D_n > d] \), associated with the n-th load, for all \( d \geq 0 \), is expressed as

\[ F_{D_n}(d) = \begin{cases} 1 & n = 0 \\ \int_0^d \int_0^d F_{Y}(y) f_{\Delta D_n}(x) dx dy & n = 1 \\ \int_0^d \int_0^d F_{Y}(y) f_{\Delta D_n}(x) dx dy & n = 2, \ldots \end{cases} \]  (5)

Here \( F_{\Delta D_n}(\cdot) \) and \( F_{Y}(\cdot) \) denote the cumulative distribution functions of random damage increments \( \Delta D_0 \) and \( \Delta D_0 \), respectively. In Eq. (5), \( F_{\Delta D_n}(d) = -dF_{\Delta D_n}(x) dx \) is the probability density function of cumulative damage to the n-th load, \( d(\cdot)/dx \) indicates the derivative with respect to \( x \). For \( n = 0 \), \( F_{\Delta D_0}(d) = \infty \), if \( d = 0 \); and \( F_{\Delta D_0}(d) = 0 \), if \( d \neq 0 \). However, \( \int_0^d \int_0^d F_{\Delta D_n}(x) dx = 1 \), in other words \( F_{\Delta D_n}(\cdot) \) is a Dirac’s delta function. The probability distributions \( F_{D_n}(d), n = 2, \ldots \), given by Eq. (5) should be obtained recursively. Fig. 1 shows schematically the form of \( f_{\Delta D_n}(\cdot) \), for a given integer value of \( n \), in which three characteristic states of damage can be identified: 1) no damage, 2) a certain level of damage and 3) collapse.

On the other hand, substituting Eq. (5) into Eq. (2), and expressing \( p[D(t) > d] \) in terms of its complement, the probability distribution function \( F_{D(t)}(d) \), Eq. (2) is rewritten as

\[ F_{D(t)}(d) = 1 - \sum_{n=1}^{\infty} p_n(t)[1 - F_{D_n}(d)] = \sum_{n=0}^{\infty} p_n(t) F_{D_n}(d) \]  (6)

According to the renewal theory [17], \( p_n(t) \) is expressed in terms of the probability distribution functions \( F_{D_n}(\cdot) \) and \( F_{S_n}(\cdot) \), which describe the time to the n-th and to the \((n + 1)\)-th load respectively, and is written as

\[ p_n(t) = \int_0^d \int_0^d \ldots \int_0^d p_n(t)[1 - F_{D_n}(d)] = \sum_{n=0}^{\infty} p_n(t) F_{D_n}(d) \]  (6)
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