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The discussion of system optimism and user equilibrium in traffic assignment with the perspective of game theory

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Abstract

In purpose of going deep into the quantitative relation between system optimism and user equilibrium, which are two classical methods of traffic assignment, the perspective of game theory is adopted. Through non-corporative game analysis to the example, it is showed that system optimism and user equilibrium can be integrated to pure strategy Nash equilibrium under infinite strategy sets. Exiting as two particular situations of Nash equilibrium, they can be converted to each other with change of the number of game players. Furthermore, the conversion process between system optimism and user equilibrium is analyzed, which is characterized by two different stages. As the number of players grows, the result of Nash equilibrium leaves away from system optimism for user equilibrium quite quickly, suggesting the inefficiency of management measures trying to achieve system optimism. The impact that parameters in impedance functions exert on process is also researched, and it is argued that the free flow travel time, which reflects the primary advantages of the route, has great influence on conversion speed and tendency.

Keywords: Traffic Assignment; Non-corporative Game; Nash Equilibrium; System Optimism; User Equilibrium

1. Introduction

Since Wardrop (1952) proposed user equilibrium (UE) and system optimism (SO), the two principles immediately became the main basis of traffic assignment algorithms. For the relationship between UE and SO, there is an agreed opinion: UE reflects the true situation where multiple travelers make decisions independently, while SO is an ideal

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state under the unified direction. Generally, the discussion on the quantitative relationship of the two is based on their own solution models. Comparatively, the mathematical model of SO is relatively more understandable, while Beckmann model (1956) used for the solution of UE is usually considered as pure mathematical methods, which is difficult to be explained. Due to the counter-intuitive nature of the latter model, although the inter-conversion between UE and SO can be realized through the modification of the travel time function (increasing the differential or integral term), it is also difficult to explain the meaning of the modification action (Lu, 2006). Therefore, this paper, based on the previous studies and with the application of the game theory, is going to analyze the relationship between UE and SO, attempting to integrate them into the same system in a broader level, so as to expand the new thoughts of traffic assignment methods.

In traffic assignment, the path choosing process of each traveler can be regarded as an independent decision-making action. However, given the restraints from capacity, different people have a mutual influence on each other and the impact is reflected by individual efficiency—change of travel time. Therefore, traffic assignment obviously is a process of game. In recent years, literature about studies on traffic assignment with game theory methods continues to appear, among which, the most typical application is taking traffic supplier and demander as the players in the game respectively to form a two-person game where the supplier applies the SO strategy while the demander adopts the UE or SUE strategy, both of them form the Stackelberg game situation with complete information trends. This type of studies can be found in refined traffic assignment and simulation (Yuan et al., 2009; An et al., 2009), traffic guidance (Ma et al., 2005; Li et al., 2009) and public transportation assignment (Sun et al., 2005). In addition, some studies are also conducted with other game methods, such as analysis on travelers' selection of difference trip modes using static game theory methods (Li et al., 2007), as well as virtualization of "path network" as the game rival of the traveler to improve the assignment algorithm (Xiao et al., 2009). Generally speaking, most of exiting researches which apply game theory are standing on the basis of two traffic assignment algorithms, namely, UE and SO, and few analyze the two classic algorithms themselves with the game theory.

It is generally believed that UE embodies individual rationality while SO reflects collective rationality. Correspondingly, non-cooperative game also shows individual rationality. Naturally it is worth to discuss whether there is equivalent relationship between the Nash Equilibrium of non-cooperative game and UE.

Some scholars have proposed that Nash Equilibrium is consistent with UE in many occasions, and Bell et al, (2000; 2002) proved it through definition. The brief discussion goes as follows: assuming that there is a simple road network with a single OD pair and N undifferentiated travelers participate in the non-cooperative game. The specific and clear road selection is a pure strategy of the traveler. The probability combination of road selection consists of a mixed strategy. Due to the homogeneity of travelers, everyone enjoys the equal probability of selection for one road, and at that time, the strategic situation formed by the person and others should have the same utilities. When N is big enough, according to the weak law of large numbers, the flow of a path j should approximately equal to the product of N (i.e. the product of the total number of travelers) and $P_{j,}$ (i.e. the undifferentiated selection probability). The same utility mentioned above also approximately equals to the time cost under such a flow. In this way, Bell established the relationship between multiple-person game utility and the travel time given certain flow. Then, it is not difficult to achieve the conclusion that Nash Equilibrium and UE by comparing their definitions.

Although Bell gave a brilliant and concise proof, such interpretation with only concepts was inevitably too abstract. Meanwhile, it is noticed that a mixed strategy with random and limited strategy set is applied to demonstrate the relationship with deterministic (in another word, nonrandom) UE. The use of both concepts of "random" and "deterministic" is prone to cause confusion in understanding. Therefore, this paper also proposes another demonstration, based on specific examples, more clear and intuitive. And in terms of methods, instead of using the mixed strategy, the pure strategy method under the unlimited strategy set is applied.

2. Pure strategy Nash Equilibrium under the unlimited strategy set

When game strategy set is a positive real number interval, it will be an unlimited strategy set. It is noticed that in the impedance function used in UE and SO assignment, the traffic flow regarded as the argument is usually taken as the continuous variables, which allows the presence of decimal places. Hence, it is also reasonable to handle the flow in the same way when analyzing the game.