Game Theory Model of a Production Resource Sharing Problem: Study of possible cheatings

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Abstract: Sharing resources could be an interesting and strategic choice for enterprises owing to their benefits. If the benefit of renting or trading are the forms of sharing that may affect enterprises of different sizes, the joint purchase and joint use can be a form of sharing interesting for SMEs which may have difficulties to buy individually production resources. However, this production resource sharing may induce possible cheatings by one of the SMEs. These cheatings, commonly deviations from the initial share's contract, are disadvantageous for the other company. The study of these possible deviations and the level of investment held by the company for the detection of deviations are then necessary. That's why, we propose in this paper to model the production resource sharing between two small enterprises and possible cheatings to evaluate them and their impact on the collaboration. The chosen model is a non-cooperative game since it allows individual profits description. Then, we compare the strategies of the game and all possible cases of cheating for each SME on the horizon of the game.

Keywords: game theories, manufacturing systems, non-cooperative game, production resource sharing, modes of cheating.

1. INTRODUCTION

Adopt an economic model, and choose to share goods, services or even knowledge between individuals and / or enterprises, is becoming increasingly popular and community [Olsen and Kemp, 2015]. Indeed, additionally to the realized economic gain from sharing, each enterprise, by contributing to the optimization of natural resources, may also improve its brand image [Dang et al, 2014]. Different kind of resources could be shared from human knowledge (R&D partnerships [Pinheiro et al, 2016], best practices and developments [Zahedi et al, 2016], etc.), to natural (for example international water sharing problems [Madani, 2010]) or "material" resources (in supply chain management [Majoub and Hennet, 2014], we can cite for example a production capacity sharing problem [Majoub and Hennet, 2010] [Roy *et al*, 2014] or production resources sharing study [Becker et al, 2016], etc.). In this paper, we will limit our study to a production resource sharing problem, for example a specific manufacturing machine.

Share resources (regardless of the type of resources) can be an interesting strategic choice for all categories of enterprises [Wu et al, 2014]. Furthermore, for small or medium-sized enterprises (SMEs), in addition to being in a situation where they do not always use their resources to a maximal rate, buying alone a new resource can be very expensive or even impossible. Then, the objective of our work is to study the opportunity for two SMEs to buy and use jointly a production resource. To study this "opportunity", we define a model

based on game theory [Von Neumann, 1944] [Boucher et al, 2010] since the considered SMEs can be seen as players, opponent in the sharing game with individual payoff affected by the decisions made by the other [Osborne and Rubinstein 1994]. Indeed, game theory is defined as a formal description of conflicting interactions between two or more actors [Myerson, 1991] [Rasmusen, 2006] and then it will allow to highlight the role play between the different entities. It has to be noticed that in our work, even if we study cooperation between two small or medium enterprises, we will use a non-cooperative game [Fudenberg and Tirole, 2000] to represent our production resource sharing problem and more precisely the individual strategy of both enterprises.

The objective of our work is firstly, to determine the different possible deviations from original contractual agreement (cheatings) achievable by each company during the joint use of the production resource, and secondly, to determine the optimal game strategies for each company during this joint use of the production resource. For this, we study and explicate in the first part the game theory model of a production resource sharing between two SMEs. Then, in the second part, we represent different modes of cheating or possible deviations for each company in the joint use of the production resource. The detection of any deviations depends on several parameters defined in the third part. We conclude our work and give some perspectives in the final part.

2. PRODUCTION RESOURCE SHARING

The commercial exchange, lease or services provider are forms of sharing economy assuming that at least one of the cooperating enterprises owns the "resource" to be shared. In this paper, we will consider a production resource, denoted M, shared between two small enterprises which cannot individually purchase this production resource. The enterprises are denoted E_1 and E_2.

The objective is to study and then to design the original contract between both enterprises. This original contract will represent the purchase and joint use of the production resource but it will also integrate the penalties sizing in case of cheating(s) to discourage it and ensure that any of both enterprises will be injured [Osborne, 1976]. More precisely, we should determine the allocated production rate of the production resource for each E_i, $i=1$ to 2, and all associated production costs for each period of share t.

It has to be noticed that the enterprises could be concurrent and produce the same type of products, or not. We only suppose that the enterprises need in their production processes the same kind of production resource, for example a particular and special machining. In a same way, we assume that both companies may have had in the past exchanges / collaboration agreements or not. Indeed, strong ties, resulting from past cooperation with very frequent exchanges, facilitate knowledge sharing and the confidence level vis-a-vis each other. At the contrary, weak ties can make collaboration more difficult by lack of trust and imply in this case to have more information on the other or close to the maximum cooperation agreement [Pinheiro et al, 2015].

The production rate for the enterprise E_i, $i=1$ to 2, denoted $\theta_i(t)$, depends on the needs of E_i, the needs of the other enterprise but also on the maintenance activities and management activities which could be changed for each period of share t. Indeed, for each period of share t, a new production rate could be set after discussion and consultation between both enterprises. Consequently, we have:

$$\theta_1(t) + \theta_2(t) + \theta_M(t) = 100\% \tag{1}$$

where $\theta_M(t)$ represents the remaining production rate at period t which must ideally be devoted to maintenance and management activities to preserve optimal performances and life duration of the machine.

Then, the joint use of the resource M allows the enterprise E_i, $i=1$ to 2, achieving a periodical profit, denoted $\pi_i(\theta_i(t),t)$, expressed as the difference between the resulting gain, denoted $G_i(t)$, which directly depends on the allocated (or real) production rate for E_i and the overall production costs paid by E_i for the given period t.

The periodical profit is given by the following equation:

$$\pi_i(\theta_i(t),t) = G_i(\theta_i(t),t) - C_i(\theta_i(t),t). \tag{2}$$

Indeed, both enterprises commit since the beginning of the contract to pay the overall production costs associated with the establishment and joint use of production resource for each period t. These costs can be fixed or variable. The production costs of enterprise E_i for a given period t, denoted $C_i(\theta_i(t),t)$, are given by:

$$C_i(\theta_i(t),t) = a_i(t).\theta_i(t) + b_i(t) \tag{3}$$

where $a_i(t)$ represents the variable production costs (maintenance and servicing activities, energy consumption, etc.) and $b_i(t)$ the fixed production costs (amortization of purchase, first commissioning, etc.) paid by E_i for a given period t.

In what follows, we detail our game theory model.

3. GAME THEORY MODEL

Our problem is represented by a non-cooperative game because it is more adapted to model the effects of decentralized decisions on network performance at tactical and operational levels [Cachon et al., 2004], [Leng et al., 2005] and since we focus on individual profit for each enterprise at each period of share, and on cheating or deviation strategies.

Indeed, at each period of share, the needs of the enterprise E_i could evolve and differ from the original period study (for example new or more important demands…) and then a real production rate, denoted $\theta_{ireal}(t)$ with $\theta_{ireal}(t) \geq \theta_i(t)$ could be really applied by enterprise E_i.

However, to simplify our study, we consider the following Assumption 1.

Assumption 1. Each enterprise maintains the same production rate (real or not) on a period of share t.

Furthermore, due for example to cash flow problems, the enterprise E_i, $i=1$ to 2, could decide not to pay all predefined production costs and then deviate from the original period agreement to only pay "real" production costs, denoted $C_{ireal}(\theta_i(t),t)$, with $C_{ireal}(\theta_i(t),t) \leq C_i(\theta_i(t),t)$. For example, in this case, the enterprise E_i, $i=1$ to 2, will not do all scheduled preventive maintenance activities but only a few of them or will do imperfect maintenance activities (not complete).

In this paper, to not have dependant stochastic variables, we also consider the following Assumption 2.

Assumption 2. At each period of share t, the enterprise E_i, $i=1$ to 2, could only deviate on the real production rate or on the production costs strictly.

We also consider an infinite horizon since the enterprises do not know when the contract will be stopped or even if it stops. Indeed, an enterprise's environment could be greatly changed over the time (due to market change for example, the demand collapse..) and / or the enterprise's contour itself can vary (modification of activities, acquisition by another group...) unpredictably and suddenly and then challenges the initial objectives and corresponding contract.

Since each enterprise, player in our game, does not know which choices will be taken by the other enterprise, the game is of imperfect memory. More precisely, at each period of time, each player knows his own use production rate of M and the corresponding production costs he has to pay, but has no information on the real production rate or on the real production costs paid by the other player.