Emergent organization in a model market

Avinash Chand Yadav [a],*, Kaustubh Manchanda [b], Ramakrishna Ramaswamy [c]

[a] *Department of Physics & Astronomical Sciences, Central University of Jammu, Samba 181 143, India*
[b] *Department of Mathematics, Indian Institute of Science, Bangalore 560 012, India*
[c] *School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110 067, India*

HIGHLIGHTS

- A market model is generalized to have certain features of real markets.
- Extremal dynamics leads to self-organization in the system.
- Observation of avalanche type features in a model market for a variety of networks.

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ABSTRACT

We study the collective behaviour of interacting agents in a simple model of market economics that was originally introduced by Nørrelykke and Bak. A general theoretical framework for interacting traders on an arbitrary network is presented, with the interaction consisting of buying (namely consumption) and selling (namely production) of commodities. Extremal dynamics is introduced by having the agent with least profit in the market readjust prices, causing the market to self-organize. In addition to examining this model market on regular lattices in two-dimensions, we also study the cases of random complex networks both with and without community structures. Fluctuations in an activity signal exhibit properties that are characteristic of avalanches observed in models of self-organized criticality, and these can be described by power–law distributions when the system is in the critical state.

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1. Introduction

Application of the methods of statistical physics and nonlinear science to different problems in economics has been an active area of research in so–called *econophysics* [1–4], prompted in part by an interest in characterizing and understanding the various mechanisms that operate in a market. By virtue of its structure, a market is a good example of an evolving complex dynamical system, being composed of a large number of interacting agents. Agents can be individuals, groups or firms; the market forms a network of agents (the nodes), and the trading forms the links, with buying and selling activities giving both direction and weight.

The network paradigm has been very useful in understanding interactions in a variety of complex dynamical systems, and the role of network topology in modifying the system dynamics has been of interest [5]. In a market, there are constraints relating to demand and supply or to available money under which each agent wishes to maximize profits. An important aspect of the study of such constrained complex systems is to understand the nature of fluctuations in the

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* Corresponding author.
  E-mail address: jnu.avinash@gmail.com (A.C. Yadav).

collective behaviour that arises from the dynamics of many interacting agents. It has been shown [6] that the distributions of different quantities such as price differences and returns have probability distributions that are non-Gaussian. The need to comprehend and characterize the mechanisms that operate in a market – in particular stochastic fluctuations, chaotic variations and nonlinearity – have seen applications of statistical mechanics to many economic models and form the basis of predictions in financial markets [6–11].

Power–law distributions in economic systems are ubiquitous, dating to the early work of Pareto [12] and studied extensively since the work of Mandelbrot [13,14]. Given the large number of interacting agents, financial markets are quintessentially complex systems that are continuously evolving. An early hypothesis for the emergence of power–laws in such systems has been that of self-organized criticality (SOC) [15–19] which has been applied extensively to various natural phenomena. Systems exhibiting SOC are characterized by slow driving and instantaneous dissipation events thus having separation of time–scales, and the system reaches its steady state, which is an attractor, without tuning of an external parameter. Applications have ranged from studies of earthquakes [20] to evolution [21], forest–fires and epidemics [22], neuronal dynamics [23] as well as to abstract entities in number theory [24]. Indeed, an early application of SOC was to the study of fluctuations in an economic model [25].

A highly simplified market model of economic behaviour with N agents on a lattice in 1–dimension (with periodic boundary conditions) was introduced by Nørrelykke and Bak (NB) [26]. The agent at site n produces q_n units of a single good at price p_n and indulges in two transactions: sells her produce to the agent at $(n-1)$ and buys the good produced by the agent at $(n+1)$; this constitutes one trading day/cycle. Every market player has a utility function given by

$$u_n = -c(q_n) + d(q_{n+1}) \tag{1}$$

where $-c(q_n)$ is the discomfort faced or utility lost in producing q_n units of goods. In this model $c(\cdot)$ is taken to be a convex function of its argument, namely increasing with increasing slope: this makes it increasingly disadvantageous for an agent to produce large quantity of goods. In contrast $d(\cdot)$, the comfort or utility gain is a concave function of its argument, typical of the principle of diminishing marginal utility.

Each agent tries to maximize the individual utility function Eq. (1) subject to the constraint

$$p_n q_n = p_{n+1} q_{n+1}. \tag{2}$$

The left side of the equation shows the total earnings of the nth agent where p_n and q_n are respectively the price per unit and the total units of goods agent n produces and wishes to sell to her neighbour at $n-1$. The right side of the equation represents the total money spent by nth agent in buying q_{n+1} units of goods produced by her neighbour at $n+1$, priced at p_{n+1} per unit. This optimization yields two quantities, namely, the level of production q_n^p and the intended consumption level q_n^w for the nth agent. Differences in the demand and supply of goods leads to each agent finally making a profit, but as trading continues, the agent with the smallest profit changes the product price in order to improve earnings. NB showed that while SOC is attained, the model has a non–stationary attractor, in contrast to the usual attracting statistically stationary critical state in most sandpile type models that show SOC [26].

In this work we extend the NB model to higher dimensions and investigate the manner in which self-organization features change. We also allow for the agents to choose, and this introduces non-local interactions in the network, and thus an additional level of complexity. Building upon the simplified model proposed by NB in one-dimension, we assume that now an agent can interact with several other agents, namely buy products from and sell goods to more than one other (see Fig. 1 below). Furthermore, agents may have different incomes, leading to differences in the level of expenditure according to the priority and capacity of each agent. This gives a weighted network with the interaction strengths differing for each link. We have examined the effect of some simple choices of weights on the system dynamics and our numerical results suggest that the SOC features of the one-dimensional Nørrelykke and Bak model carry over to higher dimensions, for regular networks as well as for complex networks including random graphs and topologies with community structures [27].

In Section 2 of this paper, we present a general framework for the NB model of interacting agents on a spatially embedded complex network. The evolution rules are also discussed there, along with a brief description of the various interaction topologies considered. The results of our simulations are presented in Section 3, which is followed by a summary and discussion in Section 4.

2. The generalized interacting market

We generalize the NB model of agents interacting by trade, namely the buying and selling of goods, as follows. Each agent continues to produce a single commodity, but this is sold to a set of customers, and goods are purchased from a set of suppliers. The number of suppliers and customers can vary from agent to agent, and clearly this forms a general directed interaction network. If the ith agent has K_i suppliers (see Fig. 1), the utility function can be written as [26]

$$u_i = -c(q_i) + \sum_{j=1}^{K_i} d_j(q_{ij}). \tag{3}$$

As discussed earlier, the functions c and d are convex and concave, respectively; we follow the earlier choices [26,28], $c(q) = q^2/2, d(q) = 2\sqrt{q}$. The first term in Eq. (3) represents the discomfort (utility lost) of the agent i in producing q_i