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Optimal effort under high-water mark contracts☆

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ABSTRACT

We mainly develop a model measuring the optimal effort of a risk-neutral hedge fund manager in a continuous-time framework. The fund manager chooses the optimal effort to maximize the present value of total fees and reduce liquidation risks, trading off extra return benefits against the cost of the effort. We find that the manager's effort depends on the ratio between the fund's assets under management (AUM) and the high-water mark (HWM), and endogenous fund liquidation has key influence on the dynamics of the effort. Our calibration suggests that when the fund is close to liquidation, the manager exerts greatest effort. The more distant the fund value is from the liquidation boundary, the less effort the manager chooses to make; when the fund value is approaching the HWM, the manager's optimal effort still decreases, but the rate of decline becomes far slower. The optimal effort contributes to both increasing the likelihood of survival for the fund and preserving the fund's going-concern value. A growth of degree of the effort cost, volatility of the AUM, exogenous liquidation probability or endogenous liquidation boundary decreases the optimal effort. We also find empirical evidence that may support our theoretical conclusion.

1. Introduction

Since the 1990s, the global hedge fund industry has developed substantially and quickly, becoming increasingly important to the modern portfolio management. According to HFR,[1] inflows and performance gains through the volatile macroeconomic environment in 2015Q1 increased total hedge fund capital to a new record of $2.94 trillion. Although the hedge fund capital posted a decline in the first quarter of 2016, it still remained above $2.87 trillion. One major feature of hedge funds is the special compensation contracts. High-water mark contracts can be regarded as the combination of option-like compensation contracts and the high-water mark (HWM), which is known as a loss carry-forward provision. Besides the management fees that are typical for mutual funds and are usually collected as 2% of the fund assets under management (AUM), i.e., the fund value, as long as the fund survives, hedge fund managers also charge performance fees. The performance fee relies on the HWM, which for each investor is the maximum value ever reached by the past fund's AUM since her investment (in some contracts, the HWM is also subject to certain adjustments). When the fund's AUM exceeds the HWM, the HWM is reset as the current fund's AUM and the manager usually receives 20% of this excess profit as a reward for good performance. In addition, the compensation contracts vary with different funds.

How is the manager's optimal effort devoted to running the fund under such compensation?

By asking this question, first, we intend to investigate the dynamics of the effort with the fund value's distance from the HWM, and discuss whether the optimal effort conduces to preserving the fund's going-concern value. Although the HWM contracts are prevalent, whether they are optimal or beneficial is an open issue. Under the principal-agent framework, Dybvig et al. (2010) discover that in a second-best world where the manager's signal about future market prices is observable but the effort is not observable, the option-like compensation contracts may be optimal. However, the research fails to describe the effort dynamics. Unlike under a management-fee compensation that is not very sensitive to the fund's performance, the manager's effort should be more dependent on the relative distance between the fund value and the HWM under the HWM contracts. Zhan (2011)

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[1] See https://www.hedgefundresearch.com.

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investigates and compares five compensation schemes that are commonly employed in the mutual fund or hedge fund industry, likewise under the principal-agent framework. Specially, the paper finds that the HWM provision induces more effort when the fund's AUM is slightly under the HWM, but it dampens the manager's effort when the fund's AUM is far away from the HWM and the manager's skill is poor. Ray and Chakraborty (2008) construct a simple optimization problem assuming that the portfolio follows uniform distribution, and find that as the distance between the fund's AUM and the HWM increases, the manager's effort falls.

However, both Zhan (2011) and Ray and Chakraborty (2008) only apply a single-period discrete-time model, assuming that hedge funds have a determined termination, and do not consider the possibility of fund liquidation during the limited period. In contrast, we study the dynamics of the managerial effort in a continuous-time framework of hedge fund valuation. Our setting is more reasonable in that the fund has a infinite horizon and can be liquidated once the exogenous or endogenous liquidation condition is triggered.[2] We find that liquidation is one crucial factor that determines the effort dynamics.

Second, this question also contributes to understanding the funds' risk-adjusted extra return, α. Manager's effort plays an important role in explaining of the fund performance. Chevalier and Ellison (1999) use the average SAT score at the fund manager's undergraduate institutions as a proxy variable to reflect the manager's effort, ability or networks of contacts, and find that mutual fund managers who attended the institutions requiring higher SAT scores have better performance. Also, they recognize that younger managers exhibit better performance than those olders, and the reason is that young managers may make more effort and work harder considering that they have longer careers ahead. Similarly, for hedge funds, although market conditions, investment opportunities and leverage choices of the fund, as well as the manager's skill and luck are all determinants of the extra return α,[3] the influence of the fund manager's own effort should not be ignored. Our findings concerning the effort can be a guide to empirical studies on hedge fund's performance.

To our knowledge, our model is the first to discuss the optimal effort of the hedge fund manager under a continuous-time framework and more practical assumptions. We extend the log-normal diffusion process setting of the AUM in Goetzmann et al. (2003) (GIR henceforth), to constant volatility σ and undetermined extra return α that the manager chooses to maximize her value function (present value of total fees). Just as GIR, throughout the paper, we assume that market opportunities, the manager's skill and the leverage are fixed, and also rule out the influence of luck on the extra return.[4] Under these assumptions, the undecided extra return can be regarded as a measure to judge the fund manager's effort, that is, α is equivalent to the manager's effort in our model. It is a natural approach, since if the manager devotes herself to information acquisition within her capacity, she would employ a better investment strategy that results in higher excess returns on the fund. In this way, we associate the fund's extra return with the effort, and measure the manager's effort explicitly.

To increase the compensation fees, the fund manager may need to seek high extra returns through great effort. At the same time, making the effort is costly. Here we take the cost function with a form similar to that of the adjustment cost function used in Hayashi (1982) and Bolton et al. (2011). So the fund manager dynamically trades off the benefit and the cost, when deciding the optimal effort α. Our model also contains other important features of hedge funds, especially the HWM contracts and fund liquidation. When the hedge fund is liquidated, the manager will lose all future payoffs, and we will show that this costly liquidation impacts the manager's optimal effort significantly.

Thanks to the fact that the risk-neutral manager's value function is homogeneous of degree one in the fund's AUM and the HWM, we find the relationship between the fund manager's effort and the ratio of the AUM and the HWM. In addition, we derive the corresponding ordinary differential equations (ODEs) for the manager's and investors' value functions, together with the lower boundary condition for liquidation and the upper boundary condition when the fund's AUM is exceeding its HWM.

In our dynamic framework, the manager's effort depends on the moneyness of the fund p, i.e., the ratio of the fund's AUM and its HWM, ranging from the lower liquidation boundary to the upper boundary, 1. The higher the moneyness p is, the lower the effort α and the decline rate. When the fund approaches liquidation, the manager tends to exert greatest effort, in spite of resulted expensive cost. The costly liquidation (downside risk) in our model damages both the manager's future payoffs and reputation. So to reduce the risk, the risk-neutral manager behaves as if she was risk-averse. Also, the degree of effort decreases with the distance of the fund from liquidation. Besides, as the fund's AUM gets closer to the HWM, the manager's optimal effort still decreases, although with a slower rate, which indicates that the manager's motivation to make the fund value exceed the HWM is not strong. The payoff of performance fees shrinks the fund's AUM, increasing the probability of liquidation in the future. Our result covers the fact that a risk-neutral manager is averse to collect performance fees too soon. However, the rate of decline is becoming slower when p approaches 1, due to the coming performance fees. The optimal effort induced in our model contributes to both increasing the likelihood of fund survival and preserving the fund's going-concern value. It can be concluded that incorporating both the HWM contracts and fund liquidation is beneficial for the survival of funds and fund investors. We also find empirical evidence that may support our theoretical conclusion.

Our findings are contrast to the results in standard settings, such as Zhan (2011) and Ray and Chakraborty (2008), since the standard framework does not involve the assumption of fund liquidation, and only focuses on the HWM contracts. The HWM contracts are thought to be a series of call options on the fund value with a changing strike price, the HWM. Therefore, it is intuitive that a risk-neutral manager makes greater effort when she is close to gaining performance fees (p is approaching 1), comparing with the effort when the option-based contracts are deep out of the money. By letting the endogenous liquidation boundary be zero (with no liquidation), our model assumption is consistent with the standard settings, and the optimal effort in this case increases as p, which agrees with the standard intuition. So we claim that liquidation is one crucial risk that could determine the effort dynamics, and the increasing effort induced by the case with no liquidation is not beneficial.

We also discuss the impacts of the effort cost, market conditions and the compensation structure. An increase in the effort cost level, volatility of the AUM, exogenous liquidation probability or endogenous liquidation boundary decreases the manager's optimal effort and total payoffs to the manager. In addition, higher rate of the management fee or performance fee induces greater effort, a higher amount of total fees paid to the manager, and a lower level of investors' value. A decrease of either the management or performance fee rate has a larger impact on all the three values than an increase, and management fees play a major role in the compensation contracts.

Our findings are related to GIR, which provide a quantitative valuation framework for hedge fund dynamics for the first time. GIR

[2] As Panageas and Westerfield (2009) (PW henceforth) mention, most hedge funds do not have a determined termination. The assumption of a infinite horizon is more reasonable.

[3] The impact of leverage choices on hedge fund's α has been widely investigated, such as Hodder and Jackwerth (2007), Panageas and Westerfield (2009) (PW henceforth), Lan et al. (2013), Buraschi et al. (2014) and Drechsler (2014). By leveraging the alpha-generating strategy, the manager can increase the extra return as well as volatility of the fund, so she must make the decision in the trade-off between the benefit and risk.

[4] The hypothesis of fixed leverage strategy is reasonable, since, just as stated in Ang et al. (2011), that the hedge fund leverage is mainly influenced by economy-wide and systematic variables and our paper does not take variant market opportunities into account.