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Evaluation of support loss in micro-beam resonators: A revisit

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ABSTRACT

This paper presents an analytical study on evaluation of support loss in micromechanical resonators undergoing in-plane flexural vibrations. Two-dimensional elastic wave theory is used to determine the energy transmission from the vibrating resonator to the support. Fourier transform and Green's function technique are adopted to solve the problem of wave motions on the surface of the support excited by the forces transmitted by the resonator onto the support. Analytical expressions of support loss in terms of quality factor, taking into account distributed normal stress and shear stress in the attachment region, and coupling between the normal stress and shear stress as well as material disparity between the support and the resonator, have been derived. Effects of geometry of micro-beam resonators, and material dissimilarity between support and resonator on support loss are examined. Numerical results show that ‘harder resonator’ and ‘softer support’ combination leads to larger support loss. In addition, the Perfectly Matched Layer (PML) numerical simulation technique is employed for validation of the proposed analytical model. Comparing with results of quality factor obtained by PML technique, we find that the present model agrees well with the results of PML technique and the pure-shear model overestimates support loss noticeably, especially for resonators with small aspect ratio and large material dissimilarity between the support and resonator.

1. Introduction

With the advent of nanotechnology, micro-electromechanical systems (MEMS) and nano-electromechanical systems (NEMS) such as micro- and nano-beam resonators have been widely used in various fields including signal processing, high precision measurement, biomedical science and basic scientific research [1,2]. Due to their ultra-high-frequency, high sensitivity and ever smaller dimensions, micromechanical resonators are suitable for wide-ranging applications such as high-frequency signal processing [3], ultra-high sensitive sensing [4,5], biological imaging [6], and even detection of single molecule [7,8] and macroscopic quantum states [9], etc.

Energy dissipation, measured by the inverse of quality factor, is one of the most important concerns in the design of MEMS/NEMS-based resonator devices, and low dissipation always leads to better performance of devices. A variety of dissipation sources have been revealed and can be classified to two categories of intrinsic loss and external loss [2,10–12], which mainly include gas damping, squeeze-film damping, thermoelastic damping, surface loss and support loss.

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support loss refers to the phenomenon that part of the vibrational energy of a resonator is dissipated to the support structure by means of elastic wave propagation. Vibration of a resonator always causes time-varying shear force and moment in the attachment region to the support, and they excite elastic waves, which transmit energy from the resonator to the support.

A great deal of work has been conducted on analysis of support loss, ranging from experimental work, theoretical analysis and finite element simulation [13–16]. As early as 1960's, Jimbo and Itao [17] derived a closed-form expression for the support loss of the fundamental mode of a cantilever resonator which is of infinite out-of-plane thickness attached to a semi-infinite medium. In the past decades, with the development of nanotechnology, support loss attracted more and more attention. Grigg and Gallacher [18] proposed an enhanced Rayleigh-Ritz-Meirovitch model for vibration analysis of the planar frame micromechanical resonators. The results of vibration analysis are then used in conjunction with the analytical model of Jimbo and Itao [17] to obtain an estimate for the support Q factor. Cross and Lifshitz [19] studied elastic wave transmission at an abrupt junction between two plates of different width but having the same out-of-plane thickness. Hao et al. [20] further studied support loss of resonators with the same out-of-plane thickness as the support modeled as a semi-infinite medium undergoing in-plane vibration, and gave a simple formula for the support quality factor. However, only the support loss due to the time-varying shear stress is considered in Ref. [20], and the shear stress is assumed to be constant over the attachment region, which is actually quadratically distributed based on Euler-Bernoulli beam theory. In addition, contribution of normal stress to the support loss and dissimilarity in material properties between the support structure and the resonator are not considered either. In general, the normal and shear stresses on the surface of the support may have a coupling effect, that is, the shear stress acting on the attachment region of the support may create normal displacement and the normal stress may produce tangential displacement. In other words, the normal stress may excite shear wave, and shear stress may excite longitudinal wave. Hao and Xu [21] also derived the expressions for the vibration displacement on substrate under several typical time-harmonic stress sources in micromechanical resonators in 2D and 3D cases, which enable the quantitative evaluation of support loss in micromechanical resonators. To the best of the authors' knowledge, almost all works on support loss available in the literature did not consider this coupling effect, which may play a non-negligible role in some situations as shown in the present study.

In order to reduce support loss, several strategies have been developed, such as employment of phononic crystal structure, reducing the clamped region and anchoring micro-resonators to the support at their nodal points [22,23].

In this study, we revisit support loss of a micro-beam resonator induced by time-varying stresses transmitted from the vibrating resonator onto the support. This problem is solved by Fourier transform and Green's function method, which results in explicit expressions of quality factor associated with the support loss. We take into account the linear distribution of normal stress and quadratic distribution of shear stress in the attachment region of the support as well as the dissimilarity in material properties of the support and the resonator. The coupling effect between shear deformation and normal deformation is considered as well. All these efforts lead to a more accurate estimation of support loss, which consists of three terms and is in explicit form as displayed later in this study. Results of quality factor obtained in this work are compared with those obtained by the previous model with only shear stress considered. Effects of beam geometry and material disparity between support and resonator are examined. Furthermore, Perfectly Matched Layer (PML) [24,25] technique is employed for validation of the present analytical model.

2. Dynamic model of micro-cantilever beam resonator

As illustrated in Fig. 1, the coordinate system is chosen in such a way that x axis is in the direction of the beam axis, y axis along the direction of thickness and z axis parallel to the width direction, and the domain of the beam resonator is defined by $0 \le x \le L$, $-b/2 \le y \le b/2$ and $-c/2 \le z \le c/2$. We hereby assume that the resonator and its support structure have the same dimension in the z direction so that the support loss in the resonator can be analyzed using a two-dimensional model. The resonator undergoes flexural vibration of small amplitude in the xy plane, which can be described by the linear Euler-Bernoulli beam theory.

The equation of motion of a micro-beam resonator in terms of deflection $Y(x,t)$ is

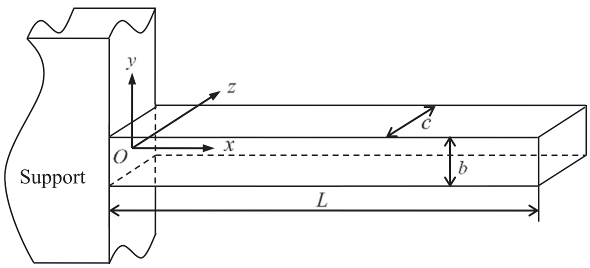

Fig. 1. Schematic illustration of a micro-cantilever beam resonator connected to a semi-infinite thin plate support structure.