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Real time trajectory optimization for nonlinear robotic systems: Relaxation and convexification ☆

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ABSTRACT

Real time trajectory optimization is critical for robotic systems. Due to nonlinear system dynamics and obstacles in the environment, the trajectory optimization problems are highly nonlinear and non convex, hence hard to be computed online. Liu, Lin and Tomizuka proposed the convex feasible set algorithm (CFS) to handle the non convex optimization in real time by convexification. However, one limitation of CFS is that it will not converge to local optima when there are nonlinear equality constraints. In this paper, the slack convex feasible set algorithm (SCFS) is proposed to handle the nonlinear equality constraints, e.g. nonlinear system dynamics, by introducing slack variables to relax the constraints. The geometric interpretation of the method is discussed. The feasibility and convergence of the SCFS algorithm is proved. It is demonstrated that SCFS performs better than existing non convex optimization methods such as interior-point, active set and sequential quadratic programming, as it requires less computation time and converges faster.

1. Introduction

Although great progresses have been made in motion planning for robotic systems [1], challenges remain in real time planning in dynamic uncertain environment. The applications include but are not limited to real time navigation [2], autonomous driving [3], robot arm manipulation and human robot cooperation [4]. To achieve real time safety and efficiency, the robot motion should be re-planned from time to time when new information is obtained during operation, which requires the motion planning algorithms to run fast enough online.

This paper focuses on optimization-based motion planning, where an ideal low level tracking controller is assumed. The method fits into the framework of model-predictive control (MPC) [5], where an optimal trajectory is obtained by solving a constrained optimization or optimal control problem in receding horizons. The optimization problem may be highly nonlinear due to the dynamic constraints, and highly non convex due to the constraints for obstacle avoidance, which makes it hard to be solved online.

Convexification [6] is a popular way to deal with non convexity by transforming the non convex problem into a convex one. One popular convexification method is the sequential quadratic programming (SQP), which approximates the non convex problem as a sequence of quadratic programming (QP) problems and solves them iteratively. References for SQP can be founded in [7] and [8]. The method has been successfully applied to offline robot motion planning as discussed in [9] and [10]. However, as SQP is designed for general purpose, the unique geometric structure of the motion planning problems is neglected. As a consequence, it is hard to use for real time applications.

In practice, the cost function for motion planning is usually designed to be convex [11,12], while the non convexity mainly comes from the physical constraints, e.g. obstacles. Regarding this observation, the convex feasible set algorithm (CFS) [13] was proposed to handle motion planning problems with convex objective functions and non convex inequality constraints. The idea of the CFS algorithm is to transform the origin problem into a sequence of convex subproblems by obtaining convex feasible sets within the non convex inequality constraints, and then iteratively solve the convex subproblems until solutions converge. The difference between CFS and SQP lies in the methods in obtaining the convex subproblems, where the geometric structure of the motion planning problem is fully considered in CFS.

However, one limitation of CFS is that it may not converge to local optima under nonlinear equality constraints (such as nonlinear dynamic constraints) as the convex feasible set for a nonlinear equality constraint may reduce to a singleton point.

In this paper, the slack convex feasible set algorithm (SCFS) is introduced to handle optimization problems with convex cost

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functions and non convex equality and inequality constraints. The idea is to relax the nonlinear equality constraints to several nonlinear inequality constraints using slack variables and then solve the relaxed problem using CFS. The feasibility, convergence and optimality of the algorithm will be proved in the paper. The performance of SCFS will be compared to that of SQP as well as other existing non convex optimization algorithms.

The remainder of the paper will be organized as follows: in Section 2, a benchmark motion planning problem is proposed; Section 3 reviews the convex feasible set algorithm; Section 4 introduces the slack convex feasible set algorithm; Section 5 illustrates the performance of SCFS; Section 6 concludes the paper.

2. Problem formulation

2.1. The notations

Denote the state of the robot as $x \in X \subset \mathbb{R}^n$ where X represents n dimensional state space. Denote the control input of the robot as $u \in U \subset \mathbb{R}^m$ where U represents m dimensional control space.[1] Suppose the robot needs to travel from x^{start} to x^{goal}. The robot trajectory is denoted as $\mathbf{x} = [x_0^T, x_1^T, \ldots, x_h^T]^T \in X^{h+1}$ where x_q is the robot state at time step q and h is the planning horizon. Without loss of generality, the sampling time t_s is assumed to be 1. Similarly, the input trajectory is denoted as $\mathbf{u} = [u_0^T, u_1^T, \ldots, u_{h-1}^T]^T \in U^h$ where u_q is the robot input at time step q. Let u_q^j denote the jth entry in u_q for $j = 1, \ldots, m$.

2.2. The benchmark problem and the assumptions

Problem 1 (*The Benchmark Problem*)**.** Consider the following optimization problem

$$\min_{\mathbf{x},\mathbf{u}} \quad J(\mathbf{x}, \mathbf{u}) \tag{1}$$

$$s.t. \quad \mathbf{x} \in \Gamma, \mathbf{u} \in \Omega, \mathcal{G}(\mathbf{x}, \mathbf{u}) = 0, \tag{2}$$

where $J : X^{h+1} \times U^h \rightarrow \mathbb{R}$ is the cost function; Γ is the constraint on the augmented state space X^{h+1}; Ω is the constraint on the augmented control space U^h; and $\mathcal{G} : X^{h+1} \times U^h \rightarrow \mathbb{R}^{mh}$ represents the dynamic relationship between states and inputs. Assumptions 2 to 6 are required.

Assumption 2 (*Cost Function*)**.** The cost function $J(\mathbf{x}, \mathbf{u}) = J_1(\mathbf{x}) + J_2(\mathbf{u})$ is smooth and bounded below by 0. J_1 is strictly convex. J_2 is strictly convex and symmetric, and it achieves minimum at $\mathbf{u} = 0$.

Assumption 3 (*State Constraint*)**.** The constraint Γ is a collection of linear equality constraints, linear inequality constraints and N nonlinear inequality constraints, i.e. $\Gamma = \cap_i \Gamma_i$ where

$$\Gamma_i = \begin{cases} \{\mathbf{x} : \phi_i(\mathbf{x}) \geq 0\} & i = 1, \ldots, N \\ \{\mathbf{x} : A_{eq}\mathbf{x} = b_{eq}\} & i = N + 1 \\ \{\mathbf{x} : A\mathbf{x} \leq b\} & i = N + 2, \end{cases} \tag{3}$$

$A_{eq} \in \mathbb{R}^{k_{eq} \times n(h+1)}$, $b_{eq} \in \mathbb{R}^{k_{eq}}$, $A \in \mathbb{R}^{k \times n(h+1)}$, and $b \in \mathbb{R}^k$. $k_{eq} < n(h + 1)$ and k are the dimensions of the constraints. $rank(A_{eq}) = k_{eq}$. Function $\phi_i : \mathbb{R}^{n(h+1)} \rightarrow \mathbb{R}$ is continuous, piecewise smooth and semi-convex, e.g. there exists a positive semi-definite matrix $H_i^* \in \mathbb{R}^{n(h+1) \times n(h+1)}$ such that for any $\mathbf{x}$, $v \in \mathbb{R}^{n(h+1)}$, $\phi_i(\mathbf{x} + v) - 2\phi_i(\mathbf{x}) + \phi_i(\mathbf{x} - v) \geq -v^T H_i^* v$. Moreover, the interior of the inequality constraints is nontrivial, i.e. $\cap_i\{\mathbf{x} : \phi_i(\mathbf{x}) > 0\} \neq \emptyset$.[2]

The linear equality constraints are for boundary conditions at the start point and the goal point. The linear inequality constraints are for state limits. The nonlinear equality constraints are for collision avoidance where ϕ_i can be identified as a signed distance function to an obstacle. The semi-convexity assumption on ϕ_i is satisfied if there is no concave corner in the obstacle.

Assumption 4 (*Control Constraint*)**.** The constraint Ω is a box constraint such that $-\bar{\mathbf{u}} \leq \mathbf{u} \leq \bar{\mathbf{u}}$ for some constant vector $\bar{\mathbf{u}} := [\bar{u}_0^1, \ldots, \bar{u}_{h-1}^m]^T > 0$ where $\bar{u}_q^j \in \mathbb{R}^+$ is the bound for u_q^j for all q and j.

Assumption 5 (*Dynamic Constraint*)**.** The dynamic equation $\mathcal{G}(\mathbf{x}, \mathbf{u}) = 0$ is affine in $\mathbf{u}$, i.e. there exist smooth functions $F : X^{h+1} \rightarrow \mathbb{R}^{mh}$ and $H : X^{h+1} \rightarrow \mathbb{R}^{mh \times mh}$ such that

$$\mathcal{G}(\mathbf{x}, \mathbf{u}) = F(\mathbf{x}) + H(\mathbf{x})\mathbf{u} = 0. \tag{4}$$

H is assumed to be diagonal, non-singular and positive definite. Eq. (4) is equivalent to

$$f_q^j(\mathbf{x}) + h_q^j(\mathbf{x})u_q^j = 0, \forall q = 0, \ldots, h - 1, j = 1, \ldots, m, \tag{5}$$

where $f_q^j : X^{h+1} \rightarrow \mathbb{R}$ and $h_q^j : X^{h+1} \rightarrow \mathbb{R}^+$ are entries in F and H, which are smooth with bounded derivatives and Hessians.

Eqs. (4) and (5) cover a wide range of typical nonlinear dynamic systems. For robot arms, let x be the robot joint position and u be the torque input, then the relationship between x and u is in the form of (5), i.e.

$$M(x_q)(x_{q+1} - 2x_q + x_{q-1}) + N(x_q, x_q - x_{q-1}) = u_q, \tag{6}$$

where $M(\cdot)$ is the generalized inertia matrix and $N(\cdot, \cdot)$ is the Coriolis and centrifugal forces. Finite differences are used to compute joint velocity and joint acceleration.

Since the input u may not be physical, the dynamic equation can also be understood in a broader sense, which is just an equation that captures the relationship between the state x and the parameter u that needs to be optimized. For example, in trajectory planning for automated vehicles, the yaw rate of the trajectory needs to be minimized. Let x be the position of the rear axle of the vehicle and u be the yaw rate, then the relationship between the yaw rate and the vehicle state assuming no tire slip ((7) below) satisfies (5),

$$(x_q - x_{q-1}) \times (x_{q+1} - x_q) = \|x_q - x_{q-1}\|^2 u_q, \tag{7}$$

where $\times$ denotes the cross product.

Moreover, the state x can also be non physical. For example, in speed profile planning for a given path [14], the state x is chosen as the time stamps along the path, while the path is sampled evenly with distance d. The input u is chosen to be the speed. The relationship between x and u ((8) below) also satisfies (5),

$$\frac{d}{x_q - x_{q-1}} = u_q. \tag{8}$$

By Assumptions 3 to 5, the constraints in (2) form a K dimensional manifold $\mathcal{M}$ where $K = n(h + 1) - k_{eq}$.[3] In order for the optimization to be nontrivial, the manifold $\mathcal{M}$ should have non empty interior, which leads us to the following assumption.

Assumption 6 (*Connected Nontrivial Domain*)**.** The domain that satisfies (2) is connected. There exist $\mathbf{x}^*$ and $\mathbf{u}^*$ that satisfy all the constraints in (2) such that $A\mathbf{x}^* < b$, $\phi_i(\mathbf{x}^*) > 0$ for all i and $-\bar{\mathbf{u}} < \mathbf{u}^* < \bar{\mathbf{u}}$.

[1] Control input u is not necessarily a physical input (such as the throttle angle for a vehicle). It can be any parameter that needs to be considered in the trajectory optimization (such as the yaw rate of a vehicle).

[2] This is to exclude the case that some combination of nonlinear inequality constraints indeed forms a nonlinear equality constraint, such as $\Gamma = \{\mathbf{x} : \phi_i(\mathbf{x}) \geq 0, -\phi_i(\mathbf{x}) \geq 0\}$.

[3] The dimension of the decision variables $\mathbf{x}$ and $\mathbf{u}$ is $n(h + 1) + mh$. As there are $k_{eq} + mh$ independent equality constraints, the dimension of the manifold is reduced to $n(h + 1) - k_{eq}$.