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Risk-averse Stochastic Nonlinear Model Predictive Control for Real-time Safety-critical Systems

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Abstract: Stochastic nonlinear model predictive control has been developed to systematically find an optimal decision with the aim of performance improvement in dynamical systems that involve uncertainties. However, most of the current methods are risk-neutral for safety-critical systems and depend on computationally expensive algorithms. This paper investigates on the risk-averse optimal stochastic nonlinear control subject to real-time safety-critical systems. In order to achieve a computationally tractable design and integrate knowledge about the uncertainties, bounded trajectories generated to quantify the uncertainties. The proposed controller considers these scenarios in a risk-sensitive manner. A certainty equivalent nonlinear model predictive control based on minimum principle is reformulated to optimise nominal cost and expected value of future recourse actions. The capability of proposed method in terms of states regulations, constraints fulfilment, and real-time implementation is demonstrated for a semi-autonomous ecological advanced driver assistance system specified for battery electric vehicles. This system plans for a safe and energy-efficient cruising velocity profile autonomously.

Keywords: Risk Assessment; Optimal Stochastic Control; Real-time Systems; Nonlinear and Optimal Automotive Control; Intelligent Driver Aids

1. INTRODUCTION

Model Predictive Control (MPC) has been an attractive approach for complex optimal control problems (Mayne, 2016). In the MPC, a constrained discrete-time Optimal Control Problem (OCP) is solved repeatedly in a receding horizon manner and the first control input in a finite sequence of control actions is applied to the system.

Uncertainty is a ubiquitous feature of complex dynamical systems, therefore, Robust MPC (RMPC) has been effectively utilised for systems with uncertainties (see e.g., Rawlings and Mayne (2012)). In addition to recently advanced formulations, early RMPC mainly was based on min-max OCP formulations. In many practical applications, however, worst-case based design may lead to conservative control actions and can result in low system performance. Stochastic MPC (SMPC) has been introduced to address the shortcomings of RMPC (see e.g., Bichi et al. (2010); Mayne (2016)). The SMPC is based on the stochastic uncertainty of a process model and generally expected value of the objective function with probabilistic constraints (chance-constraints) is optimised (Mesbah et al., 2014). Furthermore, Nonlinear MPC (NMPC) is distinguished by the use of nonlinear system models for prediction in order to improve performance specifications. Stochastic NMPC (SNMPC) utilises probabilistic descriptions of uncertainties such as parametric uncertainties, uncertain initial conditions, and exogenous disturbances to deal with the stochastic nonlinear systems (see e.g., Mesbah et al. (2014)).

While mostly there is no exact solution to the SNMPC problems. Several approximations have been developed to obtain a feasible solution rather than an exact solution (see e.g., Kantas et al. (2009)). Although these may not provide the optimal control performance, it make the SNMPC tractable in practice. Several works of literature about risk-neutral SNMPC have been developed (see e.g., Mesbah et al. (2014)). The main drawback of the SMPC is the risk-neutral expectation assessment of future random outcomes. This may not be a proper control policy for safety-critical systems where one desires to regulate the control actions robust enough to uncertainties (Yang and Maciejowski, 2015). Risk-sensitive control law of finite time for linear systems have been formulated (see e.g., Ito et al. (2015); Yang and Maciejowski (2015)). On the other hand, most of the SNMPCs depend on computationally expensive algorithms and few approaches have been devel-

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oped about risk-averse SNMPC (see e.g., Ma et al. (2012)), as well as real-time capable SNMPC (see e.g., Ohtsuka (2004); Ohsumi and Ohtsuka (2011)).

The main contribution of this paper is a real-time risk-averse SNMPC to enhance a safety-critical Advanced Driver Assistance System (ADAS) for Battery Electric Vehicles (BEV). First, the risk-sensitive Stochastic OCP (SOCP) is adapted by a log-expectation of the exponentiated quadratic performance index. Second, in contrast to RMPC which plan for the worse case, an individual expected trajectory (scenario) is generated by a physical-statistical model of uncertainty to achieve a computationally tractable design. This helps to integrate knowledge about the disturbances and propagate the uncertainty in a bounded set. Next, the risk-sensitive SOCP is converted into a certainty equivalent OCP based on the minimum principle where the scenario is considered in a risk-averse manner. The obtained approximately equivalent problem leads to a two-point boundary-value problem based on a continuation method that can be solved in real-time. The main idea of proposed method emphasises on early detection and reduction of large recourse, rather than the compensation of non-optimal decisions. Finally, the proposed approach is evaluated in terms of states regulation, constraints fulfilment, and real-time capability for a BEV specific Semi-autonomous Ecological ADAS (SEADAS). A speed prediction model based on traffic and road geometric information is utilised to estimate an expected optimistic scenario and determines probabilist velocity profile of preceding vehicle in traffic. It is shown that the proposed system is capable of improving the safety and efficiency of the BEVs that are enduring limited cruising range.

The rest of this paper is organised as follows: The risk-averse SNMPC, uncertainty quantification, and real-time algorithm for approximated SNMPC are reviewed in section 2. The concept of the SEADAS for the BEVs with risk-averse SNMPC formulation are introduced in section 3, followed by numerical evaluations in section 4. Conclusions and future research are given in section 5.

2. STOCHASTIC PREDICTIVE CONTROL

A general nonlinear system to be controlled with disturbance is described by:

$$\dot{x} = f(x, u, \omega), \tag{1}$$

$$z = h(x), \tag{2}$$

$$\omega = \Delta(z(\cdot)), \tag{3}$$

where $x \in \mathbb{R}^{n_x}$ denotes the state vector, $u \in \mathbb{R}^{n_u}$ represents the input vector, $z \in \mathbb{R}^{n_s}$ refers to the output vector, and disturbance, $\omega \in \mathbb{R}^{n_\omega}$ is random variable vector. The $\Delta(\cdot)$ is an operator standing for unmodelled dynamics that maps the output sequence over the interval $(-\infty, t]$ into ω (Mayne, 2016).

A general multi-stage SNMPC program has the form:

$$\underset{\mu}{\text{minimise}} \quad J_N(x_t, \boldsymbol{\mu}) := \mathbf{E}[\phi(x_N^*(t)) + \sum_{i=0}^{N-1} \mathcal{L}(x_i^*(t), u_i^*(t))\Delta\tau(t)] \tag{4a}$$

$$\text{subject to:} \quad x_{i+1}^*(t) = x_i^*(t) + f(x_i^*(t), \mu_i^*(t), \omega_i(t))\Delta\tau(t), \tag{4b}$$

$$\mathbf{Pr}[h_j(x_i^*(t)) \le 0] \ge \beta, \quad j = 1, \dots, q, \tag{4c}$$

$$x_0^*(t) = x(t), \quad x_i^*(t) \in \mathcal{C}, \quad x_N^*(t) \in \mathcal{C}_N, \quad \boldsymbol{\mu} \in \mathcal{U}, \tag{4d}$$

where $\mathbf{E}[\cdot]$ is the mathematical expectation and $x_i^*(t)$ denotes the state vector trajectory along the prediction τ axis. The $\boldsymbol{\mu} := \{\mu_0(\cdot), \mu_1(\cdot), \dots, \mu_{N-1}(\cdot)\}$ is an N-stage feedback control policy, which control input $u_i = \mu_i(\cdot)$ is selected at the ith stage. The $\mathcal{L} : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}_+$, and $\phi : \mathbb{R}^{n_x} \to \mathbb{R}_+$ are the cost-per-stage and the terminal cost functions, respectively. The $\mathbf{Pr}$ denotes probability and $h_j : \mathbb{R}^{n_x} \to \mathbb{R}$ is inequality constraint functions. The $\beta \in (0, 1) \subset \mathbb{R}$ denotes the confidence level (e.g., 0.9, 0.95, or 0.99), lower bound for probability $h_j(\cdot) \le 0$ that should be satisfied and q is the total number of inequity constraints. The $\mathcal{C}$ is the states constraint, and $\mathcal{C}_N$ is the terminal constraint sets. The $\omega_i(t) \in \mathbb{R}^{n_\omega}$ are composed i.i.d. random variables within a sample space Ω, a set of events (σ-algebra) $\mathcal{F}$, and $\mathcal{P}$ which is the allocations of probabilities to the events (exogenous information). The $\mathcal{U} \subset \mathbb{R}^{n_u}$ denotes the set of input constraints (for more details see Mesbah et al. (2014)). The prediction horizon, T, is divided into N steps where $\Delta\tau(t) := T(t)/N$. Given the initial state, $x_0^*(t) = x(t)$, the finite sequence of control policy, $\{\mu_i^*(t)\}_{i=0}^{N-1}$, is optimized at each sampling interval and the first control policy, $\mu_0(t)$, is applied to the system.

2.1 Uncertainty Quantification

Incorporating information about the uncertainty in parameters and variables can support in quantifying the uncertainty used in SOCP. The sources of uncertainty in various context emerge in mathematical models and experimental measurements such as parameter uncertainty, structural uncertainty, experimental uncertainty, and etc. Major uncertainty quantification problems deal with various uncertainty propagation methods such as Monte Carlo methods (Kantas et al., 2009), polynomial chaos expansion (Mesbah et al., 2014), Bayesian approaches (Yang and Maciejowski, 2015), etc. In addition, SOCP (4) mainly considered in various dynamic programming approaches such as Markov chain decision processes. Assuming Markovian structure, the emphasis generally is in identifying finite state and action sets. However, as the prediction horizon length increases in SNMPC and due to the increased number of possible scenarios with large state spaces, Markov approaches can lead to being a computationally expensive method (see e.g., Bichi et al. (2010)). Moreover, it is known that the prediction quality of these methods worsens within prediction horizon (see e.g., Mesbah et al. (2014); Schmied et al. (2015)).

Different models generally have various evaluation costs and fidelities, where high-fidelity models are usually more accurate but computationally expensive than the low-fidelity models. For practical applications, it is a reasonable approach to replace every random variable by their expected values, which lead to a more simple certainty equivalent NMPC (so-called expected value problem), or several deterministic programs, where each solution is corresponded to one particular scenario (see e.g., Ohsumi and Ohtsuka (2011)). Typically, scenarios can be obtained based on information over the random variables comes from historical data (Birge and Louveaux, 2011). The two classical reference scenarios are the expected value of the random variable and the worst-case scenario. In this paper, however, to refine the bounds on the random variable vector in SNMPC, a dynamic optimistic scenario