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Structural changes in economic growth models ⋆

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Abstract: The paper is devoted to analysis of one-sector growth models and corresponding control problems on optimal distribution of investments. The paper considers a model with a linear production function, which takes into account the feasibility of structural changes in an economy. By introducing dummy variables one can statistically indicate a period when the model undergoes changes. This provides the possibility to switch the model in different modes for providing more accurate forecasts of economic development. For the optimal control problems, the qualitative analysis of the Hamiltonian systems is implemented and solutions are constructed for each model mode. Continuous gluing of the obtained trajectories is obtained as a solution of the optimal control problem with different model modes on the infinite time interval. Comparison of the resulting model trajectory with statistical data reveals that the simulated trends provide sufficiently accurate matching with the real data. Adaptation of model parameters to a new economic mode can be considered as a *learning process* for the entire optimal control model. It makes the model more flexible with respect to the qualitative changes influencing forecasts of economic development.

Keywords: optimal control problem, economic growth, Hamiltonian trajectories, econometric data analysis, dummy variables, switching model modes

1. INTRODUCTION

Nowadays, investigation of economic processes and phenomena is one of the most significant problems which demands an interdisciplinary approach to the complex systems analysis. In this connection. it is worth to mention fundamental works of the famous economists and mathematicians proposing the methodology for constructing mathematical models which describe interconnection of the most significant macroeconomic factors, in particular, works by K. Arrow, L.V. Kantorovich, R. Sollow, K. Shell, G.M. Grossman, E. Helpman, R. Ayres, M. Intriligator, L. Krushvits and U.Ph. Sharp [Arrow (1985); Grossman and Helpman (1991); Solow (1970); Shell (1969); Ayres et al. (2009)].

Mathematical design is a basis for the statement of an optimal control problem, which is aimed for dynamic optimization of investments in increasing effectiveness of basic production factors [Crespo Cuaresma et al. (2010, 2013); Krasovskii and Tarasyev (2008b); Tarasyev and Watanabe (2001); Sanderson et al. (2010)].

It should be noted that, typically, an economic growth model has the scalability property. Therefore, it can be applied for investigation of macroeconomic aggregate factors and also for analysis of regional processes of the economic development. Moreover, a number of factors influencing dynamics of economic development of a country (or a region) is determined by their significance and it can be increased, when required, as it is proposed in the papers Tarasyev and Watanabe (2001); Krasovskii and Tarasyev (2008b); Tarasyev and Usova (2010). Significance of any factor can vary over time periods, and this fact leads, in particular, to structural changes in the model.

In this paper, the model identification is performed using methods of econometrics [Ayvazyan (2010)]. The identification procedure is focused on production functions of the linear type with switching modes. These functions include dummy variables for revealing possible qualitative changes in the economy of a country. Detection of the structural changes and their treatment in dynamic optimization methods is one of the goals of this research. Based on calibrated models, we consider control problems on optimal distribution of investments in the capital stock of country's economy. The quality of the control process is estimated by the integral consumption index discounted on the infinite time interval [Aseev and Kryazhimskiy (2007); Krasovskii and Tarasyev (2008a); Tarasyev and Usova (2010)]. The problem analysis provided in the paper is based on the Pontryagin maximum principle [Pontryagin et al. (1962)] for the problems with the infinite time horizon [Aseev and Kryazhimskiy (2007); Krasovskii et al. (2008)]. Qualitative analysis of the Hamiltonian systems within the maximum

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principle includes searching stationary regimes and the optimal investment levels which are capable to bring the economy in the domain of the favorable macroeconomic development.

The paper includes numerical results for forecast scenarios of development of the Russian economy, constructed based on the solution of the optimal control problem with linear production functions in the presence of structural changes. In conclusion, the comparison of statistical trends, econometric forecasts and simulated trajectories is provided.

The next section of the paper is devoted to the description of the growth model. Further, the econometric data analysis with dummy variables is performed for identifying model parameters. In the third section, we formulate the optimal control problem for the model of economic growth with switching modes generated by dummy variables and provide its analysis within the Pontryagin maximum principle. Numerical solutions and their comparison with statistic and econometric forecasts are carried out in the last section.

2. GROWTH MODEL

2.1 Production function

The model operates with two production factors: capital stock $K = K(t)$ and labor $L = L(t)$, which determine the output $Y = Y(t)$ by means of a production function $Y = F(K, L)$. A *production function* (or *output function*) is a functional relationship between the output Y and production factors such as capital K, labor L and *etc*.

In the model, it is assumed that production Y depends on capital K and labor L in a linear way

$$Y(t) = \alpha K(t) + \beta L(t), \quad \alpha > 0.$$

The homogeneity property of the production function allows to introduce new relative variables, which are the capital $k(t) = K(t)/L(t)$ and the production $y(t) = Y(t)/L(t)$ per one labor unit. In new variables the production function has the form

$$y(t) = \alpha k(t) + \beta = f(k(t)). \tag{1}$$

2.2 Model dynamics

The dynamics of capital K is derived using classical approaches proposed in Solow (1970)

$$\dot{K}(t) = S(t) - \mu K(t), \tag{2}$$

where the function $S(t)$ determines investments in capital $K(t)$, positive scalar μ is a depreciation rate of the capital stock. Investment $S(t)$ is a part of the output and can be represented as $S(t) = u(t)Y(t)$, where function $u(t)$ is an output share invested in capital. In the model, the parameter $u = u(t)$ plays role of a *control variable.*

The investment share $u = u(t)$ satisfies restrictions

$$0 \leqslant S(t) < Y(t) \;\Rightarrow\; 0 \leqslant \frac{S(t)}{Y(t)} < 1 \;\Rightarrow\; 0 \leqslant u(t) < 1.$$

Assumed that there exists a constant parameter $\overline{u} < 1$ determining the maximum investment level [1], *i.e.*

$$0 \leqslant u(t) \leqslant \overline{u} < 1, \quad u \in \mathcal{U} = [0, \overline{u}]. \tag{3}$$

[1] In numerical experiments maximum investment $\overline{u}$ level is estimated by the value of 0.43 according to the used data set.

It is supposed that the labor dynamics satisfies an exponential growth law

$$\dot{L}(t) = nL(t), \quad n > 0, \tag{4}$$

where nonnegative parameter n is a growth rate of the labor.

Remark: Assumption on exponential growth of the labor takes place according to the data of the Russian economy for the period from 1990 to 2013 years (see FSSS (2015)). The estimate value of the parameter n from the data is $n^* = 0.0015$.

Using dynamics of capital K (2), labor L (4) and production function (1), one can derive dynamics of relative capital $k = k(t)$

$$\dot{k}(t) = u(t)f(k(t)) - \delta k(t), \quad k(0) = k_0 = K(0)/L(0). \tag{5}$$

where positive constant $\delta = \mu + n$ denotes the level of capital depreciation rate caused by the capital amortization and growth of the labor force [2].

2.3 Balance equation

Under the assumption on the closedness of the economic system, when output $Y(t)$ can be spent on investments $S(t)$ and consumption $C(t)$, the *balance equation* can be represented in the form

$$Y(t) = S(t) + C(t) = u(t)Y(t) + C(t). \tag{6}$$

From the balance equation one can derive the consumption $C(t)$ per one worker

$$c = \frac{C}{L} = (1 - u)\frac{Y}{L} = (1 - u)y = (1 - u)f(k). \tag{7}$$

2.4 Quality of control process

The quality of the control process is estimated by the integrated consumption index of the logarithmic type. The utility theory postulates that a logarithmic function determines relative growth of a factor (in this case, consumption) in a time period

$$J(\cdot) = \int_0^{+\infty} e^{-\rho t} \ln c(t) dt, \tag{8}$$

where relative consumption level $c = c(t)$ can be found by the formula (7). Discount factor ρ is estimated by the value of 0.11 in the analyzed data.

3. ECONOMETRIC ANALYSIS

Statistical data on macroeconomic indicators of the Russian economy is chosen for the period from 1991 to 2013 FSSS (2015) (see Table 1). In general, independent variables in regression models have continuous domains. However, statistical methods for the model identification do not impose restrictions on regressors' behavior. Specifically, some variables can be discrete. Dummy (or discrete) variables describe qualitative features of the model.

Based on statistical data of the Russian economy for the period from 1991 to 2013, it is shown that in 1997 trends of macroeconomic indicators undergo changes. For revealing

[2] Basing on the data parameter δ is taken at the level of 14.185% for the model analysis.