Applied Mathematics and Computation 329 (2018) 239–250

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A genetic algorithm for the characterization of hyperelastic materials

J.R. Fernández [a,*], J.A. López-Campos [b], A. Segade [b], J.A. Vilán [b]

[a] Departamento de Matemática Aplicada I, Universidade de Vigo Escola de Enxeñería de Telecomunicación, Campus As Lagoas Marcosende s/n, Vigo 36310, Spain

[b] Departamento de Ingeniería Mecánica, Máquinas y Motores Térmicos y Fluídos Escola de Enxeñería Industrial, Campus As Lagoas Marcosende s/n, Vigo 36310, Spain

ARTICLE INFO

Keywords:
Hyperelasticity
FE analysis
Non-linear problem
Genetic algorithms

ABSTRACT

This work deals with the characterization of a hyperelastic material and the subsequent validation in different stressed states. The well-known three-parameter Mooney–Rivlin model is chosen for the sake of simplicity. In order to obtain the mechanical properties of this material, a specimen is tested using tensile forces. Once the tests are performed, the material constants are determined using a genetic algorithm to fit the experimental curve. An accurate fitness function is defined and the procedure to obtain the generations is described. Finally, with the aim to model and validate the results in a more complex setting, physical tests are performed in combination with numerical studies and FEM simulations.

1. Introduction

Hyperelastic constitutive laws are used to model materials that respond elastically when subjected to very large strains. They account both for nonlinear material behaviour and large shape changes. The main applications of the theory are to model the rubbery behaviour of a polymeric material, and to model polymeric foams that can be subjected to large reversible shape changes (e.g. a sponge).

Even if this kind of materials is widely used in industry, the difficulties to obtain an accurate mathematical characterization limit the possibilities to generate feasible numerical models that predict their behaviour. As a consequence, the virtual Finite Element (FE) models using hyperelastic materials are in some cases unable to reach accurate results. Even if nowadays there exist a good number of models (see, e.g., [1,2,5,7,8,17–19,21]), theirs material constants usually are not simple to be obtained. The simple test performed in order to find the material data is a uniaxial tensile test [24]. Once the test is carried out, the obtained results are fitted by a three-parameter Mooney–Rivlin model [16,23] using a fully incompressible assumption for the material [1,15]. The first challenge in this work is to get an accurate model characterization for a complex material using only simple tests on a specimen. Once the characterization is obtained, it will be used in a FE code in order to solve a more complex problem involving different stress states.

In order to solve characterization problems, several different methods are available to find the value of parameters in hyperelastic models. Most of authors use least square fitting methods [13], very often based on iterative processes which

* Corresponding author.
E-mail address: jose.fernandez@uvigo.es (J.R. Fernández).

https://doi.org/10.1016/j.amc.2018.02.008
0096-3003/

require an initial data to initialize the method and also differentiation of the objective function. In our case, we use Genetic Algorithms (GA) to perform the searching of parameters in order to characterize the material. This technique does not require differentiation and, although there are other available and more sophisticated methods which do not require it [26], GA are methods oriented to global optimization and are less sensitive to initialization as other techniques. Moreover, using GA we do not need to provide an initial data for the parameters to run the algorithm, although a parametric domain must be defined. Finally, using this method we are able to include multi-objective optimization if needed (for example in order to use different tests for the characterization) in a very simple way.

Genetic Algorithms represent a highly efficient search procedure for the determination of global extrema of multivariable functions, imitating the patterns of genetic reproduction in living organisms. They were first introduced by Goldberg [10] and Holland [11] and differ from conventional search algorithms in the following aspects: (1) they do not work directly with the parameter set; rather, they manipulate strings of characters representing the parameters themselves, (2) they consider many points in the search space simultaneously, (3) they use random choice to guide their search, and (4) they require no derivative information. During the last twenty years, a large number of papers have been published dealing with problems solved with this type of algorithms (see, e.g., [6,9,12,14,20,22,25,27,29] and the numerous references cited therein). They work by successive modifications of a collection (often referred to as population) of parameter combinations in the search space, using a binary coded representation termed chromosome. The initial set of chromosomes evolves through a number of operators which will be detailed in the next sections, so that subsequent generations produce more and more chromosomes in the neighbourhood of the optimum, defined through a given figure of merit, or fitness, which should be previously stated.

In this work, in order to obtain the constitutive parameters of the Mooney–Rivlin model, a genetic algorithm is used to find theirs suitable values and the better fitness as possible with the material model. Once this is fitted and the hyperelastic behaviour is reproduced, new different tests are performed, based on the characterization done, and a more complex FE model is built. The new problem involves stresses in several directions (not only uniaxial tensile stresses) and the obtained characterization will be used in a FE code to solve a large deformation problem.

The paper is outlined as follows. In Section 2, the classical three-parameter Mooney–Rivlin model is presented. Then, in Section 3 the genetic algorithm implemented for the calculation of these parameters is described, providing details concerning the population initiation, the genetic codification, the selection operators and the fitness evaluation. The results obtained with this algorithm are shown in Section 4, including a basic example to show the accuracy with a known solution and a real case. Finally, the data obtained with the second example are used in Section 5 for the simulation of the deformation of a rubber plate under extensional forces.

2. The Mooney–Rivlin model

In the classical linear elasticity theory used to model metallic behaviour at small strains, the constitutive relation between stress and strain remains linear. This assumption could be right when talking about metals suffering from stresses under their yield stress but, when talking about rubbers, foams or, for example, biological tissues, the relationship between stress and strain takes a more complex form. In the case of elastomers, strong non-linearities are observed in their stress-strain curve. Hence, the classical elasticity theory is not fulfilled by such materials and a different way to model their behaviour must be used.

In this framework, the strain energy density function is defined in order to model the hyperelastic behaviour. This function is understood as the area below the stress-strain curve, plotted in Fig. 1 as the red line, and so the shaded zone represents it.

The aim of a hyperelastic model is to obtain a formula which reproduces the strain energy density function, and after this, the constitutive law can be obtained for each elastomer material by using the large deformation elasticity theory. The definition of the strain energy density function, also called elastic potential, is given as a function of the invariants of the right Cauchy tensor.

There exist a large number of hyperelastic models (see, for instance, [1]) which can be used with different number of material constants, depending on how it is interesting to take into account the invariants of the Cauchy tensor, and the quantity and quality of the data obtained from the material to be modelled.

In this work we deal with the Mooney–Rivlin models (see, e.g., [1]), because they are one of the most widely used hyperelastic models, especially to model rubbers. Among all available Mooney–Rivlin models, the three-parameter model is the chosen one because it is a simple model which takes into account the two first invariants of the Cauchy tensor and also their product. Although models with more constants could be used, the resulting data are probably not enough to obtain a good quality of the solution.

The hyperelastic equation used to fit the obtained data must be defined according to the FE software which will be used (ANSYS). In this way, the strain energy density function is defined exactly as detailed in ANSYS user's manual and so, once the parameters are obtained, they can be used immediately in the software. It has the following form:

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + C_{11}(\bar{I}_1 - 3)(\bar{I}_2 - 3) + \frac{1}{d}(J-1)^2,$$

where W denotes the strain energy density function, C_{10}, C_{01} and C_{11} are the constants that the algorithm must approximate and d is the compressibility parameter. In this case, since the material is assumed incompressible, this term is neglected. Finally, $\bar{I}_1$ and $\bar{I}_2$ are the first two invariants of the right Cauchy–Green deformation tensor defined by Bonet [4].