Computers & Industrial Engineering 119 (2018) 114–120

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

# Greedy heuristic algorithm for packing equal circles into a circular container

Mao Chen*, Xiangyang Tang, Ting Song, Zhizhong Zeng, Xicheng Peng, Sanya Liu*

*National Engineering Research Center for E-learning, Central China Normal University, 430079 Wuhan, China*

ARTICLE INFO

*Keywords:*
Heuristics
Circle packing
Combinatorial optimization
Heuristic enumeration

ABSTRACT

This paper presents a greedy heuristic algorithm for solving the circle packing problem whose objective is to pack a set of unit circles into the smallest circular container. The proposed algorithm can be divided into two stages. In the first stage, a greedy packing procedure is introduced to determine whether the given set of circles can be packed into a fixed container. According to the greedy packing procedure, the circles are packed into the container one by one and each circle is packed into the container by a corner-occupying placement with maximal global benefit. In the second stage, the greedy packing procedure is embedded in a heuristic enumeration strategy to find the smallest container to accommodate all given circles. Tested on two sets of 20 public benchmark instances, the proposed algorithm achieves competitive results compared with existing algorithms in the literature. Furthermore, the effects of important parameter setting and essential components of the proposed algorithm are analyzed.

## 1. Introduction

Circle packing problem is a classical combinatorial optimization problem that is concerned with finding a good arrangement of multiple circles with equal or unequal radii in one or more large containers as to maximize the material utilization and hence to minimize the waste. There are various versions for this problem, e.g., the container can be circle, rectangle or polygon. This paper addresses the problem of packing equal circles into a circular container (PECC), which consists of packing $N$ unit circles $C_i$ $(i = 1, \dots, N)$ into the smallest containing circle without overlap. PECC problem is encountered in numerous real-world applications, such as production and packing for the textile, glass, automobile and aerospace industries (Hifi & M'hallah, 2009), and has been extensively studied in the literature.

The approaches for the PECC problem can be classified into two main categories. The first one tries to find optimal solutions for the problem or demonstrate the optimality of some packing patterns. However, finding a provable optimal packing is a challenging task even for very small $N$. Up to now, proven optimal packing have been found only for $N \le 10$ (Pirl, 1969), $N = 11$ (Melissen, 1994), $N = 12$ (Fodor, 2000), $N = 19$ (Fodor, 1999) and $N = 13$ (Fodor, 2003). Due to the difficulty of this approach, finding some best possible packing solutions without giving optimality proofs for them is the only feasible means to address this kind of problem.

The second category aims at proposing good solutions in reasonable time. In some studies, the problem of PECC is formulated as continuous optimization problem and some global optimization methods are proposed. Iterating among several formulations until no further improvement can be obtained by local searches, a nonlinear programming (NLP) method called reformulation descent can find solutions close to the best known for $N$ up to 100 (Mladenović et al., 2005). In Grosso, Jamali, Locatelli, and Schoen (2009), a global optimization strategy, i.e., Monotonic Basin Hopping strategy is proposed to find the global optima, which further improved many packing solutions in the range of $66 \le N \le 100$. Inspired from the physical world, some simulation approaches can obtain very good solutions for large-sized problem instances, such as quasi-physical algorithm (Huang, Li, & Xu, 2001; Wang, Huang, Zhang, & Xu, 2002; Zhang & Deng, 2005; Huang & Ye, 2011) and Billiard Simulation (BS) (Graham, Lubachevsky, Nurmela, & Ostergglrd, 1998). Indeed, the PECC problem can also be classified as a discrete optimization problem, and some combinatorial optimization algorithms that are efficient in solving unequal circle packing problem have been used or adapted for the PECC problem. However, the packing solutions by this kind of approach are not as good as those obtained by global optimization methods. For example, A1.5, a very efficient deterministic algorithm based on Maximal Hole Degree (MHD) rule for packing unequal circles into a circular or rectangular container, is "ill fitted" for the PECC problem (Huang, Li, Li, & Xu, 2006). By combining MHD rule and tree search strategy, several variants of A1.5 can improve the performance to some extent (Akeb, Hifi, & M'Hallah, 2009, 2010).

In this paper, we proposed a greedy heuristic algorithm (GHA) for solving the PECC problem. Instead of evaluating the current candidate corner-occupying placements only by MHD rule, we redefine the evaluation criterion which takes both the *hole degree* and *distance to origin*

* Corresponding authors.
*E-mail addresses:* chenmao@mail.ccnu.edu.cn (M. Chen), lsy5918@mail.ccnu.edu.cn (S. Liu).

https://doi.org/10.1016/j.cie.2018.03.030
Received 4 May 2017; Received in revised form 26 July 2017; Accepted 19 March 2018
Available online 20 March 2018
0360-8352/ 

of current placements into consideration (see Section 3.2). The proposed heuristic also differs from the algorithms in Huang et al. (2006) and Akeb et al. (2009, 2010) in terms of search strategy (see Section 3.3). All the improvements make the approach more efficient than previous MHD-based works on the PECC problem, being able to get better results within reasonable time.

The remainder of the paper is organized as follows: Section 2 presents the mathematical formulation of PECC problem, and some preliminaries related to the notion of MHD rule are presented. Section 3 describes the main components of the improved heuristic algorithm. Section 4 presents the comprehensive computational results and comparisons with some other algorithms in literature. The effects of some important parameter setting and components of the proposed algorithm are analyzed in Section 5. Section 6 summarizes the contributions of this paper and gives suggestions for future research.

## 2. Problem formulation and related works

### 2.1. Problem formulation

We first consider a decision problem stated as follows: take the origin of two-dimensional Cartesian coordinate system at the central point of the circular container of fixed radius $R$ (see Fig. 1). The coordinates of the center of the $i$th unit circle are denoted by $(x_i, y_i)$. The problem is to determine whether there exist $2N$ real numbers $x_1, y_1, \dots, x_i, y_i, \dots, x_N, y_N$ such that

$$\sqrt{x_i^2 + y_i^2} \leqslant R-1, \quad i = 1,2,\dots,N, \tag{1}$$

$$\sqrt{(x_i-y_j)^2 + (y_i-y_j)^2} \geqslant 2, \quad i,j = 1,2,\dots,N,\ i \neq j. \tag{2}$$

Constraint (1) denotes that no circle goes beyond the container boundary. Constraint (2) requires that no circle overlaps another circle in the container.

Obviously, if we can find an efficient algorithm to solve this decision problem for a fixed container, we then can solve the PECC problem by using some search strategies (e.g. dichotomous search or enumeration search, see Section 3.3) to find the minimal radius of the circular container. In the following discussion, we will first concentrate on the decision problem of fixed container.

### 2.2. Related works

In this section, some preliminaries about MHD rule are briefly introduced as follows. More detailed description can be found in Huang et al. (2006).

**Definition 1** (*Configuration*). A configuration is a pattern (layout) where $m$ ($m \geq 2$) circles have been already packed inside the container without overlap, and $N$-$m$ circles remain to be packed into the container.

If all circles have been packed into the container without overlap, the resulting configuration is called to be successful. Conversely, if some circles outside the container cannot be packed into the container anymore, the configuration is said to be failure.

**Definition 2** (*corner-occupying placement (COP)*). Given a configuration, a COP is the placement of circle $c_i$ into the container so that $c_i$ does not overlap any other previously packed circle and is tangent with two circles in the container (one of them may be the container itself). A COP for $c_i$ is denoted by $(i, x, y)$, which means $c_i$ can be placed at position $(x, y)$.

Fig. 1 illustrates a configuration where circle $c_1$, $c_2$ and $c_3$ have been placed inside the container and $c_4$ is to be packed. $P_4 = \{p_4^1, p_4^2, p_4^3, p_4^4\}$, the set of candidate COPs for $c_4$, is denoted by the dotted circles in

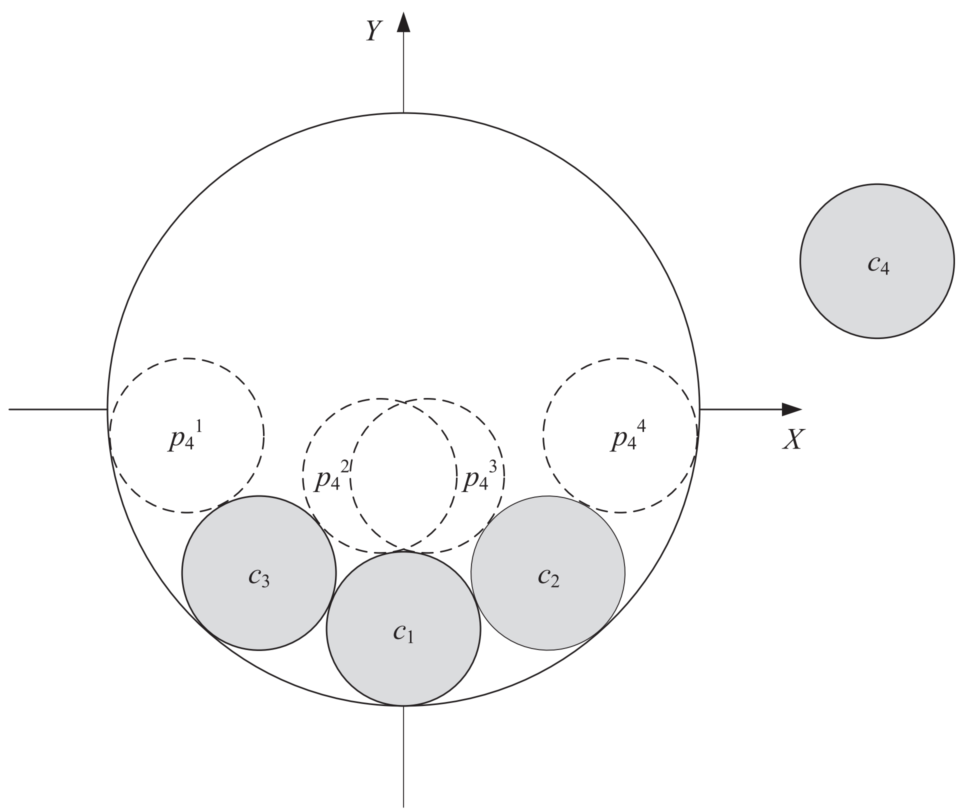


**Fig. 1.** Candidate corner-occupying placements for circle $c_4$.