Computers and Operations Research 000 (2017) 1–11

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Scheduling with job-splitting considering learning and the vital-few law

Changchun Liu, Chenjie Wang, Zhi-hai Zhang, Li Zheng*

Department of Industrial Engineering, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 22 September 2015
Revised 15 February 2017
Accepted 15 February 2017
Available online xxx

Keywords:
Parallel machine scheduling
Makespan
Job splitting
Learning effect
Vital-few law
Worst-case analysis

ABSTRACT

This research, which is motivated by real cases in labor-intensive industries where learning effects and the vital-few law take place, integrates learning and job splitting in parallel machine scheduling problems to minimize the makespan. We propose the lower bound of the problem and a job-splitting algorithm corresponding to the lower bound. Subsequently, a heuristic called SLMR is proposed based on the job-splitting algorithm with a proven worst case ratio. Furthermore, a branch-and-bound algorithm, which can obtain optimal solutions for very small problems, and a hybrid differential evolution algorithm are proposed, which can not only solve the problem, but also serve as a benchmark to evaluate the solution quality of the heuristic SLMR. The performance of the heuristic on a large number of randomly generated instances is evaluated. Results show that the proposed heuristic has good solution quality and calculation efficiency.

1. Introduction

This study examines parallel machine scheduling with job-splitting and learning effect to minimize the makespan. In this scheduling problem, a job denotes a production order, which is composed of a number of discrete identical items. An order is completed only after all the items within are finished. A job can be divided into sub-jobs to be processed on parallel machines, thus resulting in reduced throughput or improved delivery time. When items from the same job are continuously processed, learning effects occur and the single item processing time is reduced. The learning effects among items of the same job are significant but are less important than those among different jobs.

This research is motivated by real cases in labor-intensive industries, such as footwear manufacturing. In footwear manufacturing, a job denotes a production order, which is composed of a number of shoes. Different jobs represent different shoe models and sewing processes. When processing a new job, workers need to adapt to the new sewing process and start a new learning process. Therefore, the learning effects within a job are significant but are ignored among different jobs. Multiple parallel workstations are located in the factories, and jobs can be divided into sections (or sub-jobs) to be processed on parallel workstations to reduce throughput time. Although managers attempt to avoid job-splitting to enhance the beneficial effects of learning, the existence of the vital-few rule (Pareto phenomena, 80/20 rule) impels job-splitting. In the 20th century (Grosfeld-Nir et al., 2007), the economist Vilfredo Pareto introduced the vital-few rule, which states that 20% of the population possesses 80% of the wealth. Subsequently, the basic principle of vital-few rule was adopted in explaining other realities. In industries, this law indicates that specific vital jobs may occupy a large percentage of the production capacity during a scheduling period. This principle is evident in footwear industries where a few popular models stand out among a large variety of shoe models. In this case, job splitting of vital orders to parallel machine is economically necessary and common.

Fig. 1 shows three different schedules for parallel machine scheduling that considers learning effects in a footwear manufactory. Such a problem is interesting and common in practice. Sch_1 which assigns items of the job to the least heavily loaded machine one by one until all items of the job are completed, aims to complete the current job as soon as possible. In Sch_2, the job can not be split, which means the job must be processed by one machine. In such a schedule, the total processing time is minimal. However, if the processing time of a job item is long, then the job completion time will be too long. We study the potential of considering learning and job splitting simultaneously to obtain Sch_3. In such a schedule, the job must be split appropriately. Fig. 1 shows three machines, e.g., M_1, M_2 and M_3, and four different styles of shoes, e.g., A, B, C and D. C_1, C_2 and C_3 denote the completion times of schedules Sch_1, Sch_2 and Sch_3, respectively. The single item processing time will decrease when items of the same shoe style come

* Corresponding author.
E-mail address: wangcj10@mails.tsinghua.edu.cn (L. Zheng).

http://dx.doi.org/10.1016/j.cor.2017.02.011
0305-0548/© 2017 Published by Elsevier Ltd.

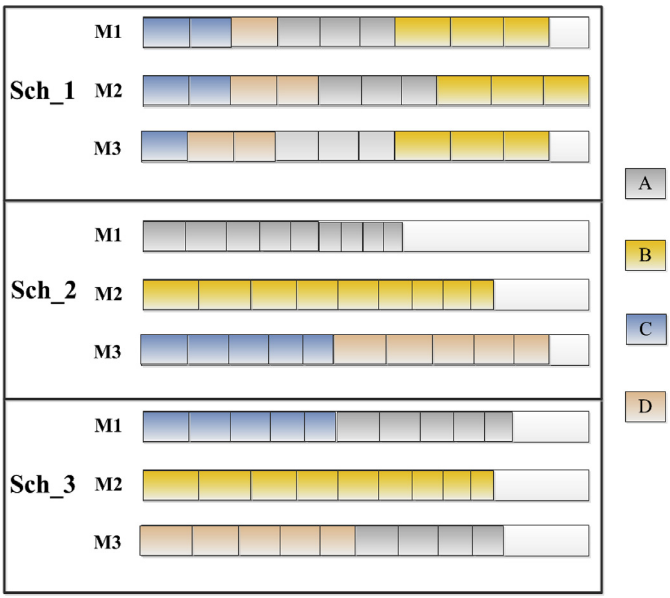

Fig. 1. A Gantt chart for a schedule for parallel machine scheduling with the consideration of learning and job splitting.

consecutively because of the learning effects. In Sch_3, Shoe A is split into two sub-jobs that are assigned to M_1 and M_3. Shoes B, C and D are assigned to machines M_2, M_1 and M_3, respectively. The comparison of Sch_1 with Sch_3 shows that the total processing time of Sch_3 is much smaller than that of Sch_1, which is influenced by the learning effects, and $C_3 < C_1$. Meanwhile, the comparison of Sch_2 with Sch_3 shows that the total processing time of Sch_2 is slightly smaller than that of Sch_3. However, the completion time of Sch_3 is much smaller than that of Sch_2, thus $C_3 < C_2$. The comparisons indicate that Sch_3, which has the shortest completion time, is the better schedule for a footwear manufactory. Therefore, studying parallel machine scheduling that considers simultaneous learning and job splitting is necessary to provide a highly efficient schedule. This paper specifically focuses on the parallel machine scheduling that aims to minimize the makespan, which is denoted as $P_m||LE, Split||C_{\max}$ according to the three-field notation system by Graham et al. (1979). The problem can easily reduce to $P_m||C_{\max}$ by setting learning rate to zero. Therefore, the problem is NP-hard. Efficient solvable algorithms are necessary because the production quantity is usually large in the labor-intensive industry.

This work has three contributions. First is the extension of the scheduling problem by considering learning and job-splitting together, which was first studied in Rudek et al. (2013). Second is the determination of the lower bound (LB) and the corresponding job-splitting algorithm for the problem. Third are developed solvable algorithms and a proven worst case ratio for the heuristic that is developed from the job-splitting algorithm. The proposed algorithms exhibit good performance on randomly generated instances.

The remainder of this paper is organized as follows. Section 2 reviews the literature on scheduling with learning and scheduling with job-splitting. Section 3 demonstrates the problem definitions and summarises the problem notations. Section 4 proposes a LB and a job splitting heuristic, which obtains the LB. Section 5 describes a heuristic developed from the job splitting heuristic, and a proven worst case ratio. Section 6 proposes a branch and bound (B&B) algorithm and Section 7 proposes a hybrid differential evolution (HDE) heuristic. Section 8 reports the experimental analysis. Section 9 presents the conclusions and possible future directions.

2. Literature review

During the last decade, numerous papers studied scheduling problems with learning effects. Biskup (1999) considered the learning effects in production scheduling and demonstrated that the single-machine scheduling problem with the learning effect remains polynomially solvable for two objectives, namely, minimizing the deviation from a common due date and minimizing the total flow-time. Extensive research has been conducted on scheduling with learning using different learning model definitions, machine environments and objectives. Moreover, extensive surveys of different scheduling models and problems involving jobs with learning effects can be found in the studies of Biskup (2008) and Janiak and Rudek (2009). Two different approaches to learning in scheduling environments were summarised by Biskup (2008). The first approach is best described as position-based learning, which denotes that learning is affected by the pure number of jobs being processed, e.g., Biskup (1999), Mosheiov (2001), Wang (2007) and Lee and Wu (2004). Alternatively, the sum-of-processing-time approach considers the processing time of all jobs processed, e.g., Kuo and Yang (2006), Koulamas and Kyparisis (2008), Rudek (2012) and Rudek (2014). More recent papers, which considered scheduling jobs with learning effects, include Wu et al. (2011), Kuo et al. (2012), Wang and Wang (2012), Li and Hsu (2012), Cheng et al. (2013), Cheng et al. (2013), Wang et al. (2013a),Wang et al. (2013b), Zhang et al. (2013) and Pan et al. (2014).

Decisions on splitting a production lot into sections and scheduling sections are discussed in the literature on scheduling with job-splitting. The splitting of parallel machines ensures that jobs can be completed as soon as possible to meet the delivery requirements. Serafini (1996) assumed that jobs may be independently split over several specified machines on the basis of the problems encountered in the textile industry. A polynomial time algorithm was provided to minimize the maximum weighted tardiness. Xing and Zhang (2000) argued that jobs can be split arbitrarily to continuous sublots, and then processed independently on m machines. They presented simple cases that were polynomially solvable and a heuristic with the worst-case performance ratio of $7/4 - 1/M$ for sequence-independent sublot setup times. Yalaoui and Chu (2003) considered a sequence-dependent sublot setup time. They transformed the problem into a traveling salesman problem, which can be efficiently solved using Little's method. Shim and Kim (2008) focused on the problem of parallel machine scheduling to minimize the total tardiness of independent jobs considering a job-splitting property. They developed a B&B algorithm to solve the problems. Wang and Wang (2012) proposed an HDE to solve the parallel machine problems with job-splitting and to minimize the makespan. Wang et al. (2016) proposed a B&B algorithm, several heuristics and greedy search for different scales of scheduling problem considering learning and job-splitting with the objective of minimizing the total completion time.

3. Problem definition and model formulation

In this paper, jobs indexed from 1 to N must be processed on machines that are indexed from 1 to M. Each job j contains N_j discrete items and can be split into several sub-jobs to be scheduled. The processing time of a single item in a job $j (j = 1, 2, \ldots, N)$ is given by p_j. Learning effects occur when items from the same job are processed continuously. The processing time of the xth repetition is expressed by function $t(x, p)$, where p is the single item processing time, x is a non-negative integer. The function $t(x, p)$ satisfies Eqs. (1) and (2),

$$t(x+1, p) \leq t(x, p), x = 1, 2, \ldots \quad (1)$$

Please cite this article as: C. Liu et al., Scheduling with job-splitting considering learning and the vital-few law, Computers and Operations Research (2017), http://dx.doi.org/10.1016/j.cor.2017.02.011