Physica A 473 (2017) 53–59

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Profit intensity and cases of non-compliance with the law of demand/supply

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HIGHLIGHTS

- We discuss the properties of the intensity of profit on markets where there are anomalies in the law of supply and demand.
- We use probabilistic interpretation of the supply and demand curves.
- We present a method of the analysis of the negative probability on markets.
- Fixed-point equation is used to find optimal strategy.

ARTICLE INFO

Article history:
Received 8 August 2016
Received in revised form 4 December 2016
Available online 3 January 2017

Keywords:
Negative probabilities
Giffen's goods
à rebours strategy
Supply and demand

ABSTRACT

We consider properties of the measurement intensity ρ of a random variable for which the probability density function represented by the corresponding Wigner function attains negative values on a part of the domain. We consider a simple economic interpretation of this problem. This model is used to present the applicability of the method to the analysis of the negative probability on markets where there are anomalies in the law of supply and demand (e.g. Giffen's goods). It turns out that the new conditions to optimize the intensity ρ require a new strategy. We propose a strategy (so-called *à* rebours strategy) based on the fixed point method and explore its effectiveness.

1. Introduction

We discuss the properties of the intensity ρ of measurement of a real random variable X with density distribution

$$pdf(p) := \frac{W(p, q = const.)}{\int_{-\infty}^{\infty} W(p, q = const.)dp}$$

restricted to domain $p \geq a$. $W(p, q)$ is the Wigner distribution of a certain quantum state for which function $W(p, q)$ is not positively defined (q and p are conjugate variables). Some of its values are negative–negative probabilities. We will examine how it affects the properties of ρ. The definition of the this functional is as follows:

$$\rho \equiv \rho(pdf, a) := \frac{-\int_{-\infty}^{-a} p \cdot pdf(p)dp}{1 + \int_{-\infty}^{-a} pdf(p)dp}. \tag{1}$$

It is naturally related to the intensity of measurements in the context of transactions (buying/selling).

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http://dx.doi.org/10.1016/j.physa.2017.01.016

One of the features of rational economic activity is to maximize profit. It is usually doing during definite intervals. The interval is chosen so that it contains period of time which is characteristic for the business activity (e.g. one month, year, a season, an insurance period or a contract date). The calculations are difficult when duration of the intervals in question is itself a random variable. To investigate activities that have different periods of duration (like in queueing theory) we define the profit intensity as a measure of this economic category. In our case, we refer to buying/selling cycle. Formula (1) represents the expectation value of the profit after the whole cycle. Here parameter a denotes the price below which a seller will always decide not to sell a given good, and the above quotient represents the rate of return in one cycle (buying/selling). It is assumed that the duration of consecutive buying and selling cycles is a random variable and quotient (1) expresses, in the language of probability density, the quotient of the average value of the profit rate in a whole cycle divided by the average duration time of this cycle. Details of the model and derivation of formula (1) are given in Section 2.

The properties of (1) were studied previously in classic and quantum market games [1–3] and in information theory context [4–6]. We envisage that the proposed approach has potential interesting applications in statistics (parameter estimation) [7].

The classical definition of a random variable implies monotonicity of its distribution function. This fact leads to an interesting property of the corresponding intensity ρ: i.e. this function has a (global) maximum at its fixed point [1]. The assumption that the function $pdf(x)$ is non-negative is used in the proof of this property. It is therefore interesting to check how extreme properties of ρ will change when this assumption is violated (the so-called negative probability). For the purpose of the discussion an old interpretation of negative probabilities is adopted. Such problems have been known since the times of Captain Robert Giffen ('40s of the 19th century [8,9]) who was the first to notice the existence of a non-monotonic market demand for the so-called Giffen goods (modern description of the discovery is the result of formulating supply/demand curves for the logarithms of prices). Giffen good is a product that is consumed more (less) of as the price rises (falls). Giffen first noticed this paradox during his observations of the purchasing habits (the potato effect or potato paradox) of the poor people in Ireland in Victorian era.

Analysis of such cases of ρ is limited to the first excited state of the quantum harmonic oscillator. This state plays a particular role in an information-theoretic measurement which entails the analysis of distributions that minimize the Fisher information function (see subjective supply and demand curves in [4,5]). The new properties of ρ that are characteristic of quantum models can, for example, be used as a test for the existence of states with negative probabilities. They also suggest a strategy for maximizing the profit earned on transactions involving Giffen goods (see Section 6).

It is worth noting that the study of negative probabilities leads to many interesting observations. This idea received increased attention in physics [10]. They are used to solve several problems and paradoxes [11,12] and also been applied to mathematical finance [13]. In this work, we propose a different look at the negative probabilities in economic context.

2. Simple market model. Profit intensity

The model presented in this section has been thoroughly studied, see [1,3–5] for more details. It is the basis for our further considerations. Let us consider the simplest possible market event of exchanging two goods which we would call asset and money and denote by Δ and \$, respectively. The proposed model comprises of two moves. First move consists in a rational buying of the asset Δ (exchanging \$ for Δ). The second move consists in a random (immediate) selling of the purchased amount of the asset Δ (exchanging Δ for \$). Let V_Δ and $V_\$$ denote some given amounts of the asset and the money, respectively. If at some time t the assets are exchanged in the proportion $V_\$:V_\Delta$ then we call the logarithmic quotation for the asset Δ the number

$$p_t \equiv \ln(V_\$) - \ln(V_\Delta).$$

If the trader buys some amount of the asset Δ at the quotation p_{t_1} at the moment t_1 and sells it at the quotation p_{t_2} at the later moment t_2 then his profit will be equal to

$$r_{t_1,t_2} = p_{t_2} - p_{t_1}.$$

Let the expectation value of a random variable ξ in one cycle (buying–selling or vice versa) be denoted by $E(\xi)$. If $E(r_{t,t+T})$ and $E(T)$, where T is the length of the cycle, are finite then we define the profit intensity for one cycle

$$\rho_t \equiv \frac{E(r_{t,t+T})}{E(T)}. \tag{2}$$

We make the following assumptions:

- The rational purchase — a purchase bound by a fixed withdrawal price $-a$ that is such a logarithmic quotation for the asset Δ, above which the trader gives the buying up. A random selling can be identified with the situation when the withdrawal price is set to $-\infty$.
- Stationary process—probability density $pdf(p)$ of the random variable p (the logarithmic quotation) does not depend on time.
- $E(p) = 0$.[1]

[1] It is sufficient to know the logarithmic quotations up to arbitrary constant because what matters is the profit and profit is always a difference of quotations.