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Partition on trees with supply and demand: Kernelization and algorithms ☆

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ABSTRACT

Network reconfiguration is an important research topic in the planning and operation of power distribution networks. In this paper, we study the partition problem on trees with supply and demand from parameterized computation perspective. We analyze the relationship between supply nodes and demand nodes, and give four reduction rules, which result in a kernel of size $O(k^2)$ for the problem. Based on branching technique, a parameterized algorithm of running time $O^*(2.828^k)$ is presented.

1. Introduction

Network reconfiguration is an important research topic in the planning and operation of power distribution networks [1,2,8,13,18,19,23,28], which is to minimize power losses or maximize load balance by intentionally partitioning a distribution network into radial subnetworks. A distribution network H can be represented by a weighted digraph $G(V, E)$ that contains a set of nodes V and a set of directed edges E. Every node represents either a "feeder" (called supply node) or a "load" (called demand node) in the distribution network, whose weight is the supply of the "feeder" or the demand of the "load". Every directed edge represents an electrical line with a switch, which can be "turned off" by the switch in the electrical line; when the switch is "turned on," the direction of the edge is the power flow direction in the electrical line. The switch must be reversed and turned on if one wishes to transport power in the other direction. The "turned off" and "reversed" operations for a switch correspond to the "edge deletion" and "edge reversion" operations in graph G respectively. The weight of an edge is the cost of the operation on its switch. A radial subnetwork T of H is denoted as a subgraph T' of G, which is a directed tree and contains at most one "feeder", and the "feeder" can supply all the power required by the "loads" in T. For simplicity, T' is called a radial subtree of G. Thus, given a weighted power graph G, the network reconfiguration problem is to do a set of edge deleting and edge reversing operations with the minimum sum of costs for "reconfiguring"

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the network or the maximum sum of all loads such that each connected component of the new graph is a radial subtree, i.e., each tree has at most one supply node, and the supply node in each tree can supply all the power required by the "loads" in the tree.

Network reconfiguration problem is proved to be NP-hard [8], which has attracted lots of attention from approximation and heuristic algorithm perspective [2,8,13,18,28]. In this paper, we study network reconfiguration problem on trees, called *Minimum Cost Partition of Trees with Supply and Demand* (MCPTSD) [17]. The problem can be modeled by a directed tree $T = (V, E)$ with node set V and edge set E, where $V = V_d \cup V_s$, and $V_d \cap V_s = \emptyset$. Each node $v \in V_d$ is called a demand node, and is assigned a positive real number $dem(v)$, called the *demand* of v. Each node $u \in V_s$ is called a supply node, and is assigned a positive real number $sup(u)$, called the *supply* of u. Each edge $e \in E$ is also assigned a positive integer $c(e)$, called the *cost* of e, which represents the cost of deleting e from T or reversing the direction of e in T. A subset $E' \subseteq E$ is called a *critical edge set*, if by deleting or reversing the edges in E', a forest $\{T_1, T_2, \cdots, T_l\}$ satisfying the following properties can be obtained:

(1) $\bigcup_{i=1}^{l} V(T_i) = V$, and $V(T_i) \cap V(T_j) = \emptyset$ $(1 \le i < j \le l)$, where $V(T_i)$ denotes the set of vertices in tree T_i;
(2) each tree T_i $(1 \le i \le l)$ contains only one supply node $u_i \in V_s$, and $\sum_{v \in V(T_i) \setminus \{u_i\}} dem(v) \le sup(u_i)$;
(3) there is a directed path from the supply node u_i to every demand node in T_i.

The definition of the MCPTSD problem is given as follows.

Given a directed tree $T = (V, E)$, where $V = V_d \cup V_s$, and $V_d \cap V_s = \emptyset$, find a critical edge set E' in T whose total cost $\sum_{e \in E'} c(e)$ is minimum for T.

Ito et al. [14] studied the problem of partitioning a tree with supply and demand such that the total obtained demand of all of subtrees is maximized, which allows that some subtrees have no supply node. They showed that the problem is NP-hard even for some particular graphs (e.g. complete bipartite graphs, a "star" with exactly one supply node). However, if the partition problem is to decide whether a given directed tree has at least one partition, without considering the objective function, it was proved in [14] that the problem can be solved in linear time. A pseudo polynomial-time algorithm and a fully polynomial-time approximation scheme (FPTAS) algorithm to solve the maximum partition problem on trees were given in [14]. In [15,16], the maximum partition problem proposed in [14] is extended to general graphs, which was proved to be MAXSNP-hard. Pseudo polynomial-time algorithms and FPTAS algorithms for series-parallel graphs and partial k-trees were also studied in [15,16]. The above mentioned approximation algorithms are restricted on some special classes of graphs (trees, series-parallel graphs). For general graphs, Popa [27] presented a $2k$-approximation algorithm, where k is the number of supply nodes. Kawabata and Nishizeki [21] gave three algorithms on trees with edge-capacities by extending the algorithms for the problems without edge-capacities in [14]. Ito et al. [17] studied the minimum partition problem on trees with minimizing the operation cost. They proved that the MCPTSD problem is NP-hard, and also gave a pseudo polynomial-time algorithm and an FPTAS algorithm for the problem. Morishita and Nishizeki [22] studied the maximum supply rate problem which find a maximum number r such that graph G has a feasible partition if the demand d_v is replaced by a new demand $d'_v = r \cdot d_v$ for every demand node v. Recently, several heuristic algorithms [19,20,24,29] have been given for the partition problem with supply and demand.

Parameterized computation is a new way dealing with NP-hard problems. For any instance (x, k) (k is the parameter) of given parameterized problem A, A is called *fixed parameter tractable* if there is an algorithm of running time $f(k)n^c$ solving instance (x, k), where c is a constant, f is a computable function. Parameterized computation has been successfully applied in many fields [5,9,10,26,30,31], and many related algorithmic techniques and methods have been developed [6,7]. In this paper, we study parameterized version of the partition problem on trees with supply and demand, which is defined as follows.

Parameterized Partition on Trees with Supply and Demand(PPTSD): Given a directed tree $T = (V, E)$, and a parameter k, where $V = V_s \cup V_d$, and $V_s \cap V_d = \emptyset$, find a critical edge set $E' \subseteq E$ with $\sum_{e \in E'} c(e) \le k$, or report that no such subset exists.

By analyzing the structure properties of the problem, we find that every demand leaf must obtain its demand through its parent node and every supply leaf either provides its supply for its parent node or not. We give three reduction rules to bound the number leaves and internal nodes of degree at least three in the reduced instance. Considering the maximum number of viable reversing edges, we present a reduction rule to bound the number of degree-2 nodes. Based on those reduction rules, a kernel of size $O(k^2)$ is given for the PPTSD problem. By studying the relationship between leaves and their neighbors, a parameterized algorithm of running time $O^*(2.828^k)$ is presented.[1]

[1] Following the convention, we use the notation $O^*(f)$ for $O(f \cdot n^{O(1)})$.

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