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Research Paper

Development of coupled numerical model for simulation of multiphase soil

K. Edip [a,*], V. Sesov [a], C. Butenweg [b], J. Bojadjieva [a]

[a] Institute of Earthquake Engineering and Engineering Seismology, Skopje, Macedonia
[b] Applied Mechanics and Structural Engineering, FH Aachen, Jülich 52428, Germany

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ABSTRACT

In this paper, a coupled multiphase model considering both non-linearities of water retention curves and solid state modeling is proposed. The solid displacements and the pressures of both water and air phases are unknowns of the proposed model. The finite element method is used to solve the governing differential equations. The proposed method is demonstrated through simulation of seepage test and partially consolidation problem. Then, implementation of the model is done by using hypoplasticity for the solid phase and analyzing the fully saturated triaxial experiments. In integration of the constitutive law error controlling is improved and comparisons done accordingly. In this work, the advantages and limitations of the numerical model are discussed.

1. Introduction

Soil as a porous medium is composed of a solid skeleton and pores, which are filled with water and/or air. In describing porous media, it is important to consider the interaction between solid, water and air phases. This interaction among the phases is particularly important in dynamic conditions such as cyclic loading. Modeling based on the porous media theory in which an average macroscopic continuum is considered has proved to be useful to better understand coupled phenomena such as flow and deformation processes. An important contribution of the paper is that it demonstrates that the numerical framework is able to consider non linearities in simulation of both flow and deformation.

Modern formulations based on multiphase mixture theories were developed in the last decade as described in details in the monograph of de Boer [1]. In order to present an introductory review of the developed theories, the Terzaghi theory can be referred as the first study of deformable porous media which dealt with the one-dimensional consolidation theory on the basis of the effective stress concept [2]. In the works of Biot [3,4] the one-dimensional theory was extended to a three-dimensional theory of consolidation. Modern mixture theories were developed based on the concept of volume fractions by Morland [5], Goodman and Cowin [6], Sampaio [7] and Bowen [8,9]. Averaging theories were developed by Whitaker [10,11] and Hassanizadeh and Gary [12–14]. The work of Biot was extended to three phase conditions with a pore air as the third phase by Fredlund and Morgenstern [15] and Chang and Duncan [16]. The theory was developed by de Boer and Kowalski [17] on material non linearity behavior of the soil skeleton in the Terzaghi Biot framework. A simple extension of the two-phase formulation of porous media considering the air pressure to be constant and equal to the atmospheric pressure was proposed by Zienkiewicz et al. [18].

A generalized incremental form which includes large deformation and nonlinear material behavior was derived by Zienkiewicz et al. [19] for liquefaction analysis of soil structures. Coupled formulations that involve both air and water phases in soils were proposed by Alonso et al. [20], Schrefler et al. [21] and Gawin et al. [22]. A general reference for the use of finite element method for the numerical simulation of fluid flow and deformation processes in porous media is the monograph by Lewis and Schrefler [23]. Recent contributions to this topic include the one in Schrefler and Scotta [24] where a two-phase flow model is used leading to two pressure variables in addition to the displacement field to be approximated by suitable finite element spaces.

The most computational aspects of coupled finite elements have been presented in both linear and nonlinearly elastic material law for fluid saturated porous solids. Wieners et al. [25] in their work consider saturated porous medium flow where the material behavior of the porous skeleton is assumed to be elasto viscoplastic. Most recently, in the work of Oettl [26] a coupled multiphase formulation is applied to geotechnical problems considering different water retention relations. The work of Holler [27] follows the

* Corresponding author.
E-mail address: kemal@pluto.iziis.ukim.edu.mk (K. Edip).

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Oettl's work in which different material models for solid phase including hypoplasticity is considered. In this particular work the development of a coupled multiphase model follows the model of Holler [27].

The model is validated against the seepage experiment of Liakopoulos [28] and partially consolidated column as given in the work of Khoei and Mohammadnejad [29]. Finally, the multiphase model is considered in the simulation of triaxial element tests in both static and dynamic conditions.

The main purpose of this paper is to present a more realistic coupled model with hypoplastic modeling of the solid state behavior of saturated and/or unsaturated soils. As for notations and symbols used in this paper, bold faced letters denote tensors and vectors; the symbol '.' denotes an inner product of two vectors (e.g. $a{\cdot}b = a_i b_i$), or a single contraction of adjacent indices of two tensors (e.g. $c{\cdot}d = c_{ij}d_{jk}$); and the symbol ':' denotes an innerproduct of two second-order tensors (e.g. $c{:}\ d = c_{ij}d_{ij}$), or a double contraction of adjacent indices of tensors of rank two and higher (e.g. $C{:}\ \varepsilon^e = C_{ijkl}\varepsilon^e_{kl}$).

2. Formulation of coupled numerical model

The numerical model containing unsaturated, or in simpler form saturated deformable porous media behavior is based on the Theory of Porous Media (TPM). This approach can be described by use of the macroscopic continuum approach proceeding from the classical theory of mixtures with additional use of the volume fraction concept. The fundamentals of porous media theories, the development and comparison with other approaches to multiphase materials can be taken from literature. The interested reader is referred to the articles by de Bowen and Ray [8,9], Bedford and Drumheller [30], de Boer and Ehlers [31], Volk and Ehlers [32] and Ricken and de Boer [33].

In porous media, the mechanical behavior of deformable porous media and the interaction among phases are considered through mass conservation, equilibrium and from the solid skeleton's constitutive behavior. In this contribution, the soil medium is considered as a porous medium composed of solid, water and air phases. Pressures of water and air phases (p_w and p_a) and displacement in two directions u_x and u_y of the solid skeleton are taken as independent variables. Local thermodynamic equilibrium is assumed and temperature is kept constant through the analyzed domain. The first quantity that is derived is the capillary pressure, which is obtained as a water to air pressure difference. Namely,

$$p_c = p_a - p_w \tag{1}$$

In partially saturated state the voids in the porous medium are filled partly by water and air. The sum of respective degrees of saturation for water saturation S_w and air saturation S_a is one:

$$S_w + S_a = 1 \tag{2}$$

Once the capillary pressure and saturation relations are known the relations between water and air saturations are evaluated by means of experimentally determined functions such as:

$$S_w = 1 - S_a = S_w(p_c) \tag{3}$$

The constitutive law of the solid phase is written in terms of the effective stress of Bishop [34] where the stress term is defined as:

$$\sigma'_{ij} = \sigma_{ij} + \delta_{ij}(S_w p_w + S_a p_a) \tag{4}$$

In Eq. (4), σ_{ij} is the total stress tensor, δ is the Kronecker symbol while p_w and p_a are the pore water pressure and the pore air pressure assuming immiscibility of the two fluids. As found by Biot and Willis [35] Eq. (4) above can be modified to account for the volumetric deformation of the soil particles in the following way

$$\sigma'_{ij} = \sigma_{ij} + \alpha\delta_{ij}(S_w p_w + S_a p_a) \tag{5}$$

where

$$\alpha = 1 - K_T/K_S \tag{6}$$

where K_T and K_S are bulk modules of the porous medium and of the solid phase, respectively. The correction induced by factor α is negligible in most soils where usually K_S is much bigger than K_T but can assume relevance when rocks are concerned.

If only small deformations are considered, the stress-strain relationship for the porous medium can be written in an incremental form:

$$d\sigma'_{ij} = D_{T,ijkl}(d\varepsilon_{kl} - d\varepsilon^0_{kl}) \tag{7}$$

where $D_{T,ijkl}$ is the fourth order tangential stiffness tensor of the material, $d\varepsilon_{kl}$ the second order tensor describing the total strain increment and $d\varepsilon^0_{kl}$ is the fraction of the strain increment not induced by the effective stress.

3. Governing equations, finite element discretization and boundary conditions

The motion of the total mixture is defined by the motion of the solid-fluid mixture and the relative motion of the fluid with respect to the mixture. In this multiphase material model description it is common to present the motion of fluids relative to the motion of the solid phase in which the velocity is described by Darcy law [14]. Thus, the fluids relative velocities for water and air phases ($\dot{\boldsymbol{u}}^{ws}$ and $\dot{\boldsymbol{u}}^{as}$) are given by

$$\begin{aligned}\dot{\boldsymbol{u}}^{ws}(x,t) &= \dot{\boldsymbol{u}}^{w}(x,t) - \dot{\boldsymbol{u}}^{s}(x,t)\\ \dot{\boldsymbol{u}}^{as}(x,t) &= \dot{\boldsymbol{u}}^{a}(x,t) - \dot{\boldsymbol{u}}^{s}(x,t)\end{aligned} \tag{8}$$

In the above Eq. (8) $\dot{\boldsymbol{u}}^{ws}$ stands for relative velocity of water phase with respect to solid phase, $\dot{\boldsymbol{u}}^{w}$ stands for velocity of water phase, $\dot{\boldsymbol{u}}^{s}$ stands for velocity of solid phase, $\dot{\boldsymbol{u}}^{as}$ represents the relative velocity of air phase with respect to solid phase and $\dot{\boldsymbol{u}}^{a}$ represents the velocity of air phase. Under the assumption of small strain theory and isothermal equilibrium, the linear momentum balance equation for the whole mixture, considering the inertial and viscosity forces, for a unit control volume can be written as [24]:

$$\sigma_{kl,k} + \rho b = \rho\ddot{u} + \varsigma\dot{u} + nS_w\rho_w[\ddot{u}^{ws} + \mathrm{grad}\dot{u}^{w}\dot{u}^{ws}] + nS_a\rho_a[\ddot{u}^{wa} + \mathrm{grad}\dot{u}^{a}\dot{u}^{as}] \tag{9}$$

where b is the body force, ς is the damping parameter which is given a small value in order to minimize the damping effects in the computations and ρ is the averaged density of the mixture which can be written as:

$$\rho = (1-n)\rho_s + nS_w\rho_w + nS_a\rho_a \tag{10}$$

In Eq. (10), n is the porosity, ρ_s, ρ_w and ρ_a are the densities of solid, water and air phases respectively. Considering the low frequency domains as given in [18] Eq. (9) can be written as:

$$\sigma_{kl,k} + \rho b = \rho\ddot{u} + \varsigma\dot{u} \tag{11}$$

The linear momentum balance equation for each fluid phase yields simply the Darcy's law

$$\dot{u}_i^{s\pi} = k_\pi(-p_{\pi,i} + \rho_\pi(b_i - \ddot{u}_i)) \tag{12}$$

where $\dot{u}$ is the velocity of the fluid phase relative to the moving solid, π is either water or air phase and k_π is the permeability relative to the πth phase which is assumed to be isotropic. The dissipative terms arising in a multiphase flow system at the interfaces are taken into account through the relative permeability $k_{r\pi}$.

$$k_\pi = \frac{k}{\mu_\pi}k_{r\pi} \tag{13}$$

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