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Modeling and simulation of viscoelastic film retraction

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ABSTRACT

In this paper, we investigate the retraction of a circular viscoelastic liquid film with a hole initially present in its center by means of finite element numerical simulations. We study the whole retraction process, aiming at understanding the hole opening dynamics both when the hole does not feel any confinement and when it interacts with the solid wall bounding the film. The retraction behavior is also interpreted through a simple toy model, that highlights the physical mechanism underlying the process.

We consider three different viscoelastic constitutive equations, namely, Oldroyd-B, Giesekus (Gsk), and Phan Thien–Tanner (PTT) models, and several system geometries, in terms of the film initial radius and thickness. For each given geometry, we investigate the effects of liquid inertia, elasticity, and flow-dependent viscosity on the dynamics of the hole opening. Depending on the relative strength of such parameters, qualitatively different features can appear in the retracting film shape and dynamics.

When inertia is relevant, as far as the opening hole does not interact with the wall bounding the film, the influence of liquid elasticity is very moderate, and the retraction dynamics tends to the one of Newtonian sheets; when the hole starts to interact with the solid wall, hole radius/opening velocity oscillations are detected. Such oscillations are enhanced at increasing elasticity. From the morphological point of view, the formation of a rim at the edge of the retracting film is observed. If inertial forces become less relevant with respect to viscous forces, *R*-oscillations disappear, the hole opening velocity goes through a maximum and then monotonically decays to zero, and no rim forms during the film retraction. Geometrical changes have the effect of enlarging or reducing the portion of the retraction dynamics not influenced by the presence of the solid wall with respect to the one governed by the hole-wall interactions.

1. Introduction

The retraction of liquid sheets is of interest in a wide range of scientific and technological fields, ranging from biological membranes to foam production [1].

The very first observations of such phenomenon were made on soap bubbles by Dupré [2] and Lord Rayleigh [3], in the nineteenth century. More than half a century later, an experimental work by Ranz [4] reported that a punctured soap film retracts under the influence of surface tension at an almost constant speed, and, during the retraction, the liquid tends to accumulate in a rim around the opening hole. About ten years later, Taylor [5] and Culick [6] independently derived a mathematical expression for such velocity, which turns out to be $u_{TC} = \sqrt{2\Gamma/(\rho h)}$, with Γ the surface tension between the liquid and the ambient fluid, ρ the liquid density, and *h* the film thickness. McEntee and Mysels [7] provided experimental validation of Taylor–Culick theory for soap films thicker than 0.1 μm, while Keller [8] extended the theory to sheets of non-uniform thickness.

At the end of last century, Debrégeas et al. [9,10] performed experiments on PDMS sheets, and found that, when the liquid is highly viscous, the retraction is much slower than the one of soap films, and does not occur at constant velocity. They also derived an exponential law for the hole radius growth, which reads $r(t) = R_0 \exp[\Gamma t/(\eta h)]$, where R_0 is the initial hole radius (as soon as the film gets punched) and η is the liquid viscosity. Moreover, no rim was observed during the retraction in PDMS films.

Brenner and Gueyffier [11] studied the retraction of a planar liquid sheet by means of the lubrication theory, finding that the rim formation depends on the relative weight of viscosity, surface tension and inertia, and identified a dimensionless parameter that gives a measure of the interplay of such effects, namely, the Ohnesorge number $\mathrm{Oh} = \eta/\sqrt{2\Gamma\rho h}$. In particular, the rim is observed at low Oh-values,

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whereas it does not appear at large Oh. Song and Tryggvason [12] performed numerical simulations that extended the work by Brenner and Gueyffier [11] by taking into account the presence of an ambient fluid surrounding the liquid sheet. Their simulations show that, when the viscosity of the ambient fluid is within 10% of the viscosity of the liquid film, the influence of the ambient fluid is negligible. Further numerical investigations were performed by Sunderhauf et al. [13], where the mechanism of rim formation when inertial or viscous effects are dominant was studied.

Recently, Savva and Bush [14] investigated through the lubrication theory the retraction of unbounded liquid sheets with planar and circular shapes, validating the results from Taylor [5], Culick [6], and Debrégeas et al. [9,10], and elucidating the effects of viscosity, geometry and initial conditions.

All the above mentioned papers, both experimental and theoretical, deal with films made of Newtonian liquids. In this paper, we study the retraction of a discoid viscoelastic liquid film with a hole initially present in its center through finite element numerical simulations. The competition between inertial and elastic effects on the phenomenon is studied. Moreover, both constant-viscosity and flow-depending-viscosity constitutive equations are investigated.

Examining the dynamics of viscoelastic liquid sheets would allow to understand the range of process parameters where liquid elasticity 'counts', which is of interest from both a scientific and a technological point of view, since many industrial problems, e.g., polymer foaming, deal with films involving viscoelastic liquids. We investigate under what process conditions the unbounded retraction velocity, i.e., the hole opening velocity when the hole does not interact yet with the bounding wall, is significantly influenced by the elasticity of the liquid with respect to the Newtonian case. The numerical results are interpreted by means of a simple heuristic model that highlights the physics underlying the retraction process. In addition, we also consider what happens when the opening hole comes in the proximity of the solid wall that bounds the retracting film.

2. Problem outline

Fig. 1 sketches the system under investigation: a discoid viscoelastic liquid film Ω with radius L and initial thickness h_0 has a concentric hole with initial radius R_0. Along the film thickness, the hole has not a uniform radius, as the latter goes from R_0, at the middle of the film, to $R_0 + h_0/2$, at the top and the bottom of the film cross section. The hole has a uniform initial curvature radius $h_0/2$. A cylindrical coordinate system is set with its origin in the center of the hole (and of the film), its r-axis oriented along the film radius, and its z-axis oriented along the film thickness. At $r = L$, the liquid film is in contact with a cylindrical solid wall.

Assuming that the system is isothermal, that the material is incompressible, i.e., the volume of the viscoelastic film is constant, and neglecting the effects of the gravity, the film dynamics is governed by the mass and momentum balance equations in the following form:

$$\nabla\cdot\boldsymbol{u} = 0 \tag{1}$$

and

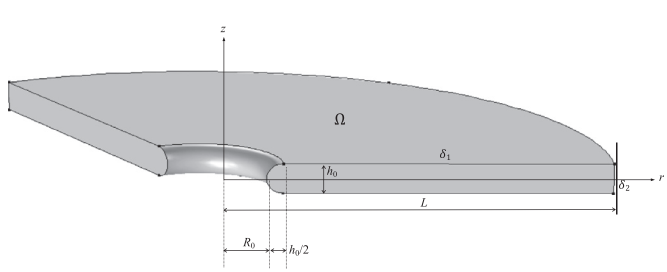

Fig. 1. Geometry of a discoid viscoelastic film with radius L and initial thickness h_0. A concentric hole is present with initial radius R_0 and curvature radius $h_0/2$.

$$\rho\frac{\partial\boldsymbol{u}}{\partial t} + \rho\boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \eta_s\nabla^2\boldsymbol{u} + \nabla\cdot\boldsymbol{\tau} \tag{2}$$

where $\boldsymbol{u}$ is the velocity vector, ρ is the density of the liquid, t is the time, p is the pressure, η_s is the 'Newtonian' contribution to the liquid viscosity, and $\boldsymbol{\tau}$ is the viscoelastic contribution to the stress tensor, respectively. Notice that, if a Newtonian liquid was considered, the last term in the rhs of Eq. (2) would disappear.

We consider three different viscoelastic constitutive equations, namely, the Oldroyd-B, Giesekus (Gsk) , and exponential Phan Thien–Tanner (PTT) models [15]. The Oldroyd-B model is widely used to describe viscoelastic liquids with a constant viscosity, and it reads

$$\lambda\overset{\triangledown}{\boldsymbol{\tau}} + \boldsymbol{\tau} = 2\eta_p\boldsymbol{D} \tag{3}$$

with $\overset{\triangledown}{\boldsymbol{\tau}} = \frac{D\boldsymbol{\tau}}{Dt} - (\nabla\boldsymbol{u})^T\cdot\boldsymbol{\tau} - \boldsymbol{\tau}\cdot\nabla\boldsymbol{u}$ the upper-convected time derivative, λ the viscoelastic liquid relaxation time, η_p the non-Newtonian contribution to the viscosity of the liquid, and $\boldsymbol{D} = \frac{1}{2}(\nabla\boldsymbol{u} + \nabla\boldsymbol{u}^T)$ the symmetric part of the velocity gradient tensor. The Giesekus constitutive equation is commonly employed to describe the rheological behavior of polymer solutions. It can be written as follows:

$$\lambda\overset{\triangledown}{\boldsymbol{\tau}} + \boldsymbol{\tau} + \frac{\lambda\alpha}{\eta_p}\boldsymbol{\tau}^2 = 2\eta_p\boldsymbol{D} \tag{4}$$

where α is the parameter that modulates the dependence of the viscosity on the strain rate. Finally, the exponential Phan Thien–Tanner constitutive equation typically models polymer melts, and reads

$$\lambda\overset{\triangledown}{\boldsymbol{\tau}} + \exp\left[\frac{\lambda\epsilon}{\eta_p}\mathrm{tr}(\boldsymbol{\tau})\right]\boldsymbol{\tau} = 2\eta_p\boldsymbol{D} \tag{5}$$

with ϵ the parameter that modulates the dependence of the viscosity on the strain rate and $\mathrm{tr}(\boldsymbol{\tau})$ the trace of the $\boldsymbol{\tau}$-tensor. Notice that when the parameters α and ϵ go to zero, both the Gsk and the PTT constitutive equations 'degenerate' into the Oldroyd-B model.

The balance equations that describe the system in Fig. 1 are supplied with the following boundary conditions:

$$\boldsymbol{T}\cdot\boldsymbol{n} = \Gamma\,\boldsymbol{n}\nabla\cdot\boldsymbol{n} \quad \text{on } \delta_1 \tag{6}$$

$$\boldsymbol{u}\cdot\boldsymbol{n} = 0 \quad \text{on } \delta_2 \tag{7}$$

$$(\boldsymbol{I} - \boldsymbol{n}\boldsymbol{n})\cdot(\boldsymbol{T}\cdot\boldsymbol{n}) = \boldsymbol{0} \quad \text{on } \delta_2 \tag{8}$$

In Eq. (6), $\boldsymbol{T} = -p\boldsymbol{I} + 2\eta_s\boldsymbol{D} + \boldsymbol{\tau}$ is the total stress tensor in the fluid, Γ is the surface tension, and $\boldsymbol{n}$ is the outwardly directed unit vector normal to the boundary. Eq. (6) is the condition on stress on the free surfaces of the film, i.e., all the surfaces not in contact with the wall. It is worth remarking that, like in [12], the fluid surrounding the liquid film is not considered in the domain, yet its presence enters the problem through the surface tension Γ in Eq. (6). Eq. (7) is the adherence condition between the liquid film and the solid wall at $r = L$ in the r-direction, whereas Eq. (8) is the perfect slip condition on the z-component of the velocity at $r = L$. Notice that Eqs. (7) and (8) imply that the length L of the domain shown in Fig. 1 is constant and the film can only slide tangentially to the wall.

We assume that, at $t = 0$, the liquid film is motionless and stress-free, namely,

$$\begin{aligned} \boldsymbol{u}|_{t=0} &= \boldsymbol{0} \quad \text{in } \Omega \\ \boldsymbol{\tau}|_{t=0} &= \boldsymbol{0} \quad \text{in } \Omega \end{aligned} \tag{9}$$

Following Savva and Bush [14], the model equations are made dimensionless with the film initial thickness h_0 as the characteristic length, $t_i = \sqrt{\rho h_0^3/(2\Gamma)}$ as the characteristic time, the Taylor–Culick velocity $u_{TC} = \sqrt{2\Gamma/(\rho h_0)}$ as the characteristic velocity, and $\hat{\tau} = 2\Gamma/h_0$ as the characteristic stress, with $\eta_0 = \eta_s + \eta_{p0}$ the zero shear total viscosity of the liquid. Notice that t_i is an 'inertial' characteristic time; actually, another characteristic time can be identified, namely, a