ARTICLE IN PRESS
JID: SIMPAT [m3Gsc;July 2, 2016;15:11]

Simulation Modelling Practice and Theory 000 (2016) 1–11

Contents lists available at ScienceDirect

Simulation Modelling Practice and Theory

journal homepage: www.elsevier.com/locate/simpat

On the optimal computing budget allocation problem for large scale simulation optimization

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ARTICLE INFO

Article history:
Received 24 October 2015
Revised 15 May 2016
Accepted 27 May 2016
Available online xxx

Keywords:
Simulation optimization
Large scale problems
Ordinal optimization
Optimal computing budget allocation

ABSTRACT

Selecting a set that contains the best simulated systems is an important area of research. When the number of alternative systems is large, then it becomes impossible to simulate all alternatives, so one needs to relax the problem in order to find a good enough simulated system rather than simulating each alternative. One way for solving this problem is to use two-stage sequential procedure. In the first stage the ordinal optimization is used to select a subset that overlaps with the actual best systems with high probability. Then in the second stage an optimization procedure can be applied on the smaller set to select the best alternatives in it. In this paper, we consider the optimal computing budget allocation (*OCBA*) in the second stage that distribute available computational budget on the alternative systems in order to get a correct selection with high probability. We also discuss the effect of the simulation parameters on the performance of the procedure by implementing the procedure on three different examples. The numerical results indeed indicate that the choice of these parameters affect its performance.

1. Introduction

In this paper we consider the following simulation optimization problem

$$\min_{\theta \in \Theta} Y(\theta) \tag{1}$$

where Θ is a feasible solution which is arbitrary, finite and large. Let Y be the expected performance measure of a certain complex stochastic system, written as $Y(\theta) = E[L(\theta, X)]$, where θ is a vector that represents system design parameters, X represents all the random effects of the system and L is a deterministic function that depends on θ and X.

The set of m best systems is defined as the set that contains all the systems with the m smallest means, which is unknown and to be inferred through simulation. Suppose that the feasible solution set contains n systems. Let Y_{ij} (observation) represent the jth sample of $Y(i)$ for system i. We assume that Y_{ij} are independent and identically distributed (*i.i.d.*) and follow a normal distribution with unknown means $Y_i = E(Y_{ij})$ and variances $\sigma_i^2 = Var(Y_{ij})$, i.e. $Y_{i1}, Y_{i2}, \ldots, Y_{iT_i}$ are *i.i.d.* $N(Y_i, \sigma_i^2)$.

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http://dx.doi.org/10.1016/j.simpat.2016.05.004

The normality assumption does not pose a problem here, as simulation outputs are obtained from batch means or through an average performance; therefore, by using the Central Limit Theorem (*CLT*) the normality assumption holds. In practice, the variance σ_i^2 is unknown, so we estimate it using the sample variance s_i^2 for Y_{ij}.

In this paper we seek to select a set of size m, S_m, such that the m best systems are contained within it. We define as best the system that has the smallest mean in the case of minimization problem. Let $T_i, i = 1, \ldots, n$ be the simulation length of system i, then let $\overline{Y}_i = \frac{1}{T_i}\sum_{j=1}^{T_i} Y_{ij}$ be the sample mean for system i. Let $\overline{Y}_{[r]}$ be the rth smallest (order statistic) of $\{\overline{Y}_1, \overline{Y}_2, \ldots, \overline{Y}_n\}$, i.e. $\overline{Y}_{[1]} \le \overline{Y}_{[2]} \le \cdots \le \overline{Y}_{[n]}$, then let $S_m = \{[1], [2], \ldots, [m]\}$. The correct selection is defined by S_m containing all of the m systems with smallest means, i.e. $CS_m = \{\max_{i\in S_m} \overline{Y}_i \le \min_{i\notin S_m} \overline{Y}_i\}$.

Most of the previous work focuses on selecting a single best system. In many cases, it is important to provide a set of good systems rather than select the best one. We consider the problem of selecting a subset of the best m out of n systems based on simulation output. If the size of the feasible solution set is small, then Ranking and Selection (*R&S*) procedures can be used to select the best system or a subset that contains the best systems, see Bechhofer et al. [1], Law and Kelton [2], Goldsman and Nelson [3], and Kim and Nelson [4–6]. (*R&S*) procedures require a great amount of computational time, therefore, for large scale problems, these procedures may not be feasible. Thus, the idea of ordinal optimization (*OO*) proposed by Ho et al. [7] came to relax the objective to finding good enough systems, rather than accurately estimate the performance value of these systems. The objective of the *OO* procedure is to isolate a subset of good systems with high probability, then any simulation optimization procedure can be used to locate the optimal solution(s) from the isolated set. Many sequential selection procedures are proposed to select a good system when the number of alternatives is large, see Almomani and Rahman [8], Alrefaei and Almomani [9], Almomani and Alrefaei [10]. All these procedures are still focused on selecting a single best system or selecting a subset containing one of the best systems. As a result, the selected subset may also contain very poor solutions, which can affect the convergence rate of the procedure. To fill this gap, instead of selecting the best system from a given set or finding a subset that is highly likely to contain the best system, the objective is modified to select a set of m systems from the actual best systems. Instead of distributing the available computing budget to all feasible solutions equally, the available computing budget can be distributed smartly so as to maximize the probability of correctly selecting the good solutions. This is the idea of the optimal computing budget allocation $OCBA_m$ procedure for selecting the best m systems, which was proposed by Chen et al. [11,12].

Almomani and Alrefaei [13] consider a sequential selection procedure for selecting a good subset of systems when the number of alternatives is very large. This procedure is a combination of the (*OO*) and the $OCBA_m$ procedures. In the first stage, the *OO* procedure is used to isolate a subset of good enough systems with high probability, therefore, it reduces the size of the search space so that it is appropriate to apply the $OCBA_m$ procedure. In the second stage, we use the $OCBA_m$ to formulate the problem as a maximization of the probability of correctly selecting all best m systems from the subset that is selected by the *OO* procedure in the first stage, subject to a constraint on the total number of simulation replications available. The procedure started by simulating the solutions in the isolated set with initial simulation replications of size t_0, then a fixed increment of the simulation replications of size Δ will be added to the solutions in the isolated set. The process is repeated until the available computing budget is consumed. In this paper, we study the effect of certain simulation parameters such as the initial sample size t_0 and the increment in simulation samples in each step, Δ, on the performance of this procedure. The procedure is applied to solve different examples, including Monotone Increasing Mean, $M/M/1$ queuing system, and buffer allocation problem, with various values of t_0, Δ and the total simulation budget, T to study their effect on the performance of the selection procedure. Furthermore, we aim to determine appropriate values of t_0 and Δ, since there is no clear formulation to determine these two values when the number of alternatives is very large.

Generally speaking, large scale optimization problems are difficult to solve using classical optimization solution methods. In many real-world applications, such problems are tackled using heuristics. For example, Diabat [14] uses a joint genetic algorithms and simulated annealing approach a vendor managed inventory system in a two-echelon supply chain. Diabat et al. [15,16], and Diabat and Deskoores [17] use different heuristics approaches for solving various joint facility location and inventory management problems. Fu et al. [18], Diabat and Effrosyni [19], Zeng et al. [20], Al-Dhaheri et al. [21,23], and Al-Dhaheri and Diabat [22] use other heuristics-based techniques to solve optimization problems arise from the maritime logistics industry.

This paper is organized as follows: Sections 2 and 3 provide the background of *OO* and $OCBA_m$ procedures. In Section 4, the sequential selection procedure is presented, while in Section 5 numerical examples are provided. Finally, Section 6 includes concluding remarks.

2. Ordinal optimization

The ordinal optimization (*OO*) procedure has been proposed by Ho et al. [7] to deal with large scale selection problems. The idea of OO is to select a system or a subset of systems from the feasible solution set that contains some of the best systems with high probability, rather than focusing on accurately estimating the values of the system performance during the optimization process. This means that order should first be considered before values are estimated, or in other words ordinal optimization precedes cardinal optimization. The objective is to isolate a subset of good enough systems with high probability and reduce the required simulation time for the discrete event simulation.