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An approximate simulation model for initial luge track design

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ABSTRACT

Competitive and recreational sport on artificial ice tracks has grown in popularity. For track design one needs knowledge of the expected speed and acceleration of the luge on the ice track. The purpose of this study was to develop an approximate simulation model for luge in order to support the initial design of new ice tracks. Forces considered were weight, drag, friction, and surface reaction force. The trajectory of the luge on the ice track was estimated using a quasi-static force balance and a 1d equation of motion was solved along that trajectory. The drag area and the coefficient of friction for two runs were determined by parameter identification using split times of five sections of the Whistler Olympic ice track. The values obtained agreed with experimental data from ice friction and wind tunnel measurements. To validate the ability of the model to predict speed and accelerations normal to the track surface, a luge was equipped with an accelerometer to record the normal acceleration during the entire run. Simulated and measured normal accelerations agreed well. In a parameter study the vertical drop and the individual turn radii turned out to be the main variables that determine speed and acceleration. Thus the safety of a new ice track is mainly ensured in the planning phase, in which the use of a simulation model similar to this is essential.

1. Introduction

Bobsled, luge, and skeleton are considerable fast winter sports, for example at the Whistler Sliding Centre a speed of 41 m/s (148 km/h)[1] and a normal acceleration of 5*g* were measured in luge. Even for elite runners such high speeds and accelerations are difficult to handle. Because of safety issues, ice tracks have to be restricted in maximum speed and maximum normal acceleration. For the initial design of new tracks one needs a knowledge of the expected running dynamics of the bobsled, luge, or skeleton. Misjudgment in the development may lead to inadmissible high speeds and, consequently, to accident prone accelerations acting on the athlete.

To simulate sport on artificial ice tracks one needs data of the track course as well as of drag and friction. General construction principles for bobsled tracks were given in Stoye (1990). Data for a particular track can be obtained from construction plans. For example the bobsled track for the 1964 Winter Olympics in Innsbruck had already an elliptic cross section and clothoids for the transition of straight to turn sections. New tracks still use these elements and usually are built as combined tracks for bobsled, luge, and skeleton.

Drag and friction have been described by several authors. Walpert and Kyle (1989) reported values for the drag coefficient in sport including luge. Brownlie (1992) investigated the effect of body segments and apparel on the drag resistance in the case of running, cycling and skiing. In many cases drag resistance data are classified by the national federations (e.g. Meile, 2006). Comprehensive descriptions of friction on snow and ice were given by Colbeck (1992) and Petrenko and Whitworth (2002). Friction of steel on ice was investigated for skating by Evans et al. (1976) and de Koning et al. (1992), for bobsled by Hainzlmaier (2005) and Itagaki et al. (1987), and for luge by Fauve and Rhyner (2008). To our knowledge, no results are available for the shearing force in the transverse direction for a runner on ice.

Simulations in the ice track sport have been done either by solving the 3d problem or by solving a simplified 1d problem. Hubbard et al. (1989) modeled the surface of the Calgary track using a bicubic spline and solved the 3d equations of motion for the bobsled on that surface. These simulations were used to develop bobsled simulators in order to support the training of athletes (Kelly and Hubbard, 2000). Günther et al. (1994) described the modeling of the steering forces in the simulation. Braghin et al. (2010) presented a 3d simulation model for the Cesana Pariol track implementing steering as well as side wall contact forces for the bobsled. When the path of the sled on the track surface is given, the 3d problem reduces to a 1d problem. Roche et al. (2008) and Larman et al. (2008) integrated the 1d equation of motion for the skeleton along the baseline of the track in St. Moritz and Kedzior

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[1] Track record, February 14, 2010.

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et al. (1988) for an arbitrarily given track. In these simulations, performance was investigated.

In the initial design of a new ice track the course of the track has to be fitted to the terrain. Because of safety issues the International Luge Federation requests for the homologization of the Sochi track a maximum speed below 37.5 m/s (135 km/h) (FIL, 2010) and the rules of the International Bobsleigh and Skeleton Federation (FIBT, 2010) require a centrifugal force below 5g. Thus, for new and as yet undesigned tracks one needs an estimate of speed and normal acceleration acting on the luge with the athlete and, consequently, the purpose of this work was to develop an approximate simulation model to predict speed and normal acceleration. For this the trajectory of the luge in the track was estimated and the 1d equation of motion was solved along that trajectory. The model was used to identify the drag area and the coefficient of friction in elite luge. For validation normal accelerations calculated by the simulation and measured during these runs were compared. Finally, the most significant variables were identified that affect the maximum speed and maximum normal acceleration for a given track.

2. Method

2.1. Model of the ice track

The Whistler ice track was chosen to test the method. The track has a nominal length of 1379 m, a vertical drop of 153 m, and 16 turns with turn radii varying from 12 to 100 m.

The track surface is given by the baseline, which follows the bottom of the track, and the cross section. The track is roughly divided into straights and turns. The latter are further divided into entrance, turn, and exit sections. For the baseline one needs altitude $h_0(s_0)$, inclination angle $\alpha_0(s_0)$, and radius $r_0(s_0)$ as a function of the distance s_0 along the baseline. In the turn sections the radius r_0 of the baseline is constant and in the transitions from straight to turn sections it is given by the radius of a clothoid with parameter A. Let s_0^s and s_0^e be the distance along the baseline of the start and the end of the entrance section of a turn. Then, for $s_0^s \le s_0 \le s_0^e$ the radius of the baseline is given by

$$r_0(s_0) = \frac{A^2}{s_0 - s_0^s}. \qquad (1)$$

Each exit section is modeled as reversal of an entrance section with a different clothoid parameter. All these data are given by the construction plan and related tables (IBG, 2004). To get a continuous 3d representation of the baseline of the track as a function of the distance s_0, the data for h, α_0, and r_0 are linearly interpolated (Fig. 1).

In straight sections the cross section is a horizontal line with the baseline in the middle. In the turn section the cross section of the track is modeled as a quarter ellipse, with half axes a and b approximating the Lillehammer track's surface (Strømme, 1991). The baseline is located at the bottom of the quarter ellipse and the inward part of the cross section is a horizontal line (Fig. 2). In the transitions the cross section is given by a horizontal line, an ellipse section, and an inclined line. The inclined line vanishes at the end of the entrance section. The ellipse parameters $a(s_0)$ and $b(s_0)$ are given by

$$a(s_0) = a\sigma^4(5-4\sigma), \quad b(s_0) = b\sigma^2(3-2\sigma), \qquad \sigma = \frac{s_0 - s_0^s}{s_0^e - s_0^s} \in [0,1]. \qquad (2)$$

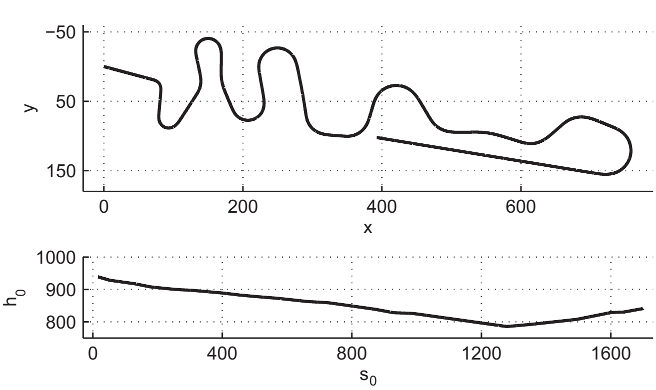

Fig. 1. Top view (coordinates x and y [m]) of the Whistler ice track (above) and altitude h_0 [m] versus distance along the baseline s_0 [m] (below).

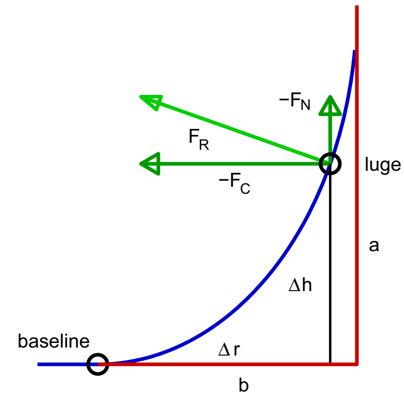

Fig. 2. Forces acting on the luge. The position of the luge is given by that point of the quarter ellipse for which the surface reaction force F_R is normal to the ellipse. Note, the cross section shown in the diagram is normal to the baseline, hence, inclined by the angle α_0 to the vertical.

Note, with the given definition of the cross sections the assumed trajectory of the luge, as explained in Section 2.3, is on the ellipse section and is an overall continuous curve.

2.2. Equation of motion

We distinguish between quantities related to the baseline (index 0) and the trajectory of the luge (no index). The 1d equation of motion is formulated along the trajectory of the luge.

The athlete with the luge is modeled as a point mass m of 117 and 205.5 kg in single and double luge, respectively. Forces considered are weight F_W, drag F_D, surface reaction force F_R, and friction F_F. Let α be the inclination angle of the trajectory relative to horizontal. Then, the luge is accelerated by the projection of the weight on the trajectory

$$F_P = mg\sin\alpha. \qquad (3)$$

Let ρ be the density of air, C_dA the drag area of the athlete with the luge, and v the speed, then the drag is given by

$$F_D = \tfrac{1}{2}\rho C_d A v^2. \qquad (4)$$

With the ideal gas law and the barometric height formula (US Gov, 1976) the density of air is given by temperature T and altitude h

$$\rho = \frac{pM}{RT}, \quad p = p_0\left(1 - \frac{Lh}{T_0}\right)^{gM/RL}. \qquad (5)$$

The Whistler track runs between 939 and 786 m above sea level. At an air temperature of 0 °C the density of air increases from 1.152 at the start to 1.175 kg/m^3 at the finish.

In straight sections the surface reaction force is given by $F_R = F_N$ with F_N the projection of the weight on the cross section of the track

$$F_N = mg\cos\alpha_0. \qquad (6)$$

In turn, including entrance and exit sections, the surface reaction force is approximately given by the norm of the vector sum of F_N and the centrifugal force

$$F_C = \frac{mv^2}{r} \qquad (7)$$

hence,

$$F_R = \sqrt{F_N^2 + F_C^2}. \qquad (8)$$

Since F_R is normal to the track surface (Fig. 2), the friction force is given by

$$F_F = \mu F_R \qquad (9)$$

with μ the coefficient of friction.

The equation of motion along the assumed path is given by

$$m\ddot{s} = F_P - F_D - F_F \qquad (10)$$

leaving the problem of establishing equations for the location Δr of the trajectory of the luge within the cross section.

2.3. Trajectory of the luge

The integration of the equation of motion requires the calculation of the turn radius r and the inclination angle α of the trajectory. In straight sections the