Mathematical Social Sciences 93 (2018) 90–100

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Partially cooperative games[☆]

Tomohiko Kawamori

Faculty of Economics, Meijo University, 1-501 Shiogamaguchi, Tempaku-ku, Nagoya 468-8502, Japan

HIGHLIGHTS

- We present a general model of interdependent bargaining situations.
- We define a solution concept for this model.
- We provide a sufficient condition for a solution to exist.
- We give a noncooperative foundation to such solutions.
- We propose a solution to the merger paradox as an application.

ARTICLE INFO

Article history:
Received 30 July 2017
Received in revised form 3 March 2018
Accepted 3 March 2018
Available online 13 March 2018

ABSTRACT

In this paper, we present a general model in which mutually dependent negotiations are simultaneously conducted and define a solution concept for the model. We provide a sufficient condition for the solution to exist and show that the solutions approximately coincide with the equilibrium outcomes of extensive form games. We present a solution to the merger paradox as an application.

1. Introduction

Interdependent bargaining situations are ubiquitous in the real world. For example, consider the following scenario:

> There are four firms, 1, 2, 3 and 4. Firms 1 and 2 form an R&D alliance and firms 3 and 4 also form an alliance. Each alliance simultaneously decides its level of R&D. Then, each firm independently decides its quantity or price.

Within each alliance, firms collectively decide their R&D level. It is usual to make collective decisions by bargaining. However, this decision making is more complex: in bargaining within each alliance, the profit frontier and disagreement point depend on the R&D level of the other alliance.[1]

In this paper, we present a general formalization of interdependent bargaining situations, a *partially cooperative game*. Each player belongs to some decision groups, each decision group chooses an alternative given a disagreement alternative, and each player's payoff is determined depending on the tuple of the alternatives chosen by the decision groups. Note that for any decision group, given an alternative tuple of the other decision groups, a bargaining problem within this decision group is naturally defined. We call an alternative tuple s^* a *Nash solution* in G if for any decision group C, the alternative of decision group C in s^* is the Nash bargaining solution to the bargaining problem within decision group C given the alternative tuple of the other decision groups in s^*. We also define a solution concept in the case where transfers are available within each decision group. We provide a sufficient condition for a Nash solution to exist. We show that Nash solutions with transfers approximately coincide with the equilibrium outcomes of extensive form games. We present a solution to the merger paradox as an application: two downstream firms respectively transact firm-specific intermediate goods with two upstream firms; by merger of the two downstream firms, the merged firm transacts two upstream firms, enjoys an advantageous bargaining position and obtains a large profit.

Our formulation includes some existing theoretical framework as special cases. In an extreme case where no player cooperates, a partially cooperative game and a Nash solution are equivalent to a strategic form game and a Nash equilibrium (Nash, 1950a, 1951), whereas in another extreme case where all players cooperate, they are equivalent to a bargaining problem and the Nash bargaining solution (Nash, 1950b, 1953). In intermediate cases, multiple players cooperate, but not all. Thus, we call G a *partially*

☆ I am grateful to two anonymous referees, Yuji Fujinaka, Hiroaki Ino, Ming Hsin Lin, Toshiji Miyakawa, Yoshihiro Ohashi, Kazuo Yamaguchi and the participants at the Conference on the Dynamics of Oligopoly and Competition and the Lunchtime Seminar at Osaka University of Economics for their valuable comments. This work was supported by KAKENHI (Grant Number 23730201 and 15K17028).

E-mail address: kawamori@meijo-u.ac.jp.

1 For further examples in addition to the above scenario, some countries engage in a negotiation on a free trade agreement (FTA), and other some countries, another FTA; in a legislature, a committee decides on a matter, and another committee, another matter.

https://doi.org/10.1016/j.mathsocsci.2018.03.001
0165-4896/

cooperative game. If every pair of players cooperates, the other groups of players do not cooperate, and each pair is given two alternatives, a partially cooperative game and a Nash solution are equivalent to a network formation game and a pairwise stable network (Jackson and Wolinsky, 1996).

Rubinstein (1982) and Binmore et al. (1986) give the Nash bargaining solution a noncooperative foundation. Our paper gives any Nash solution a noncooperative foundation in a similar way. That is, we define extensive form games as follows: a player proposes an alternative, and the other players respond to the proposal by accepting or rejecting it; if all responders accept it, the proposal is implemented, and otherwise, the procedure is repeated. In our extensive form games, proposal–response procedures in multiple coalitions are simultaneously conducted.

Okada (2010) investigates the relationship between the Nash bargaining solution and stationary subgame perfect equilibria in coalitional bargaining games. In his extensive form games, the coalitions form endogenously, whereas in our extensive form games, they are exogenously given. In Okada (2010), only one negotiation is conducted in each period, whereas in our paper, multiple negotiations may be simultaneously conducted. This difference results in the difference in the uniqueness of the equilibrium outcome: in Okada (2010), the uniqueness holds, whereas in our paper, it may not hold.

Genicot and Ray (2006) investigate a situation where there are a principal and multiple agents, the principal bilaterally contracts with each agent, and each agent's payoff by her outside option is increasing in the number of uncontracted agents. Genicot and Ray (2006) consider a noncooperative game such that once an agreement is achieved, the contract remains binding in subsequent periods. Genicot and Ray (2006) show that the principal simultaneously contracts with some agents and sequentially contracts with the other agents later; by doing so, she can exploit the agents. The situation (underlying environment) in Genicot and Ray (2006) is a special case of the situation of our paper. However, the noncooperative game (bargaining procedure) is different from that of our paper, in which an agreement is binding in one period. Thus, the results are different.

Bennett (1997) formulates interdependent bargaining situations, defines a solution, shows the existence of the solution and gives the solution a noncooperative foundation. In Bennett (1997), a player's payoff at the disagreement point within a coalition depends on only agreements in the other coalitions that she belongs to, whereas in our paper, it depends on agreements in the other coalitions that she does not belong to as well as belongs to. Moreover, in Bennett (1997), the set of attainable payoff tuples for a coalition does not depend on agreements in the other coalitions, whereas in our paper, it does depend.

Chakrabarti et al. (2011) present a game where some players cooperate and show existence of equilibria. In Chakrabarti et al. (2011), some players are noncooperative and the other players are cooperative among themselves; this situation is, by our paper's words, described as each noncooperative player forms a singleton decision group and cooperative players jointly form a single decisive group whose alternatives are the their action tuples. In Chakrabarti et al. (2011), disagreement alternatives are not introduced, and cooperative players are assumed to maximize the sum of their payoffs with respect to their action tuples, whereas in our paper, players in each decision group bargain over their alternatives.

In the literature, several papers study interdependent bargaining situations in particular circumstances: e.g., negotiations between an upstream firm (supplier) and a downstream firm (buyer) in Horn and Wolinsky (1988) and Chipty and Snyder (1999); negotiations between a firm and a union in Davidson (1988) and Zhao (1995); negotiations between a supplier of a public good and a consumer of it in Matsushima and Shinohara (2015).[2] The present paper provides them a unified viewpoint.

Horn and Wolinsky (1988) and Chipty and Snyder (1999) are related to our application on a solution to the merger paradox.

Horn and Wolinsky (1988) investigate the situation where upstream and downstream firms negotiate prices of intermediate goods, and subsequently downstream firms decide the quantities of final goods. They compare three cases: (i) two pairs of a upstream firm and a downstream firm simultaneously bargain; (ii) a single upstream firm bargains with two downstream firms respectively; (iii) a upstream firm and a downstream firm bargain. They show that the profitability of merger depends on whether the differentiated final goods are substitute or complementary. A result related to our application is that if the final goods are substitute, merger of downstream firms is not profitable. In Horn and Wolinsky (1988), only prices of intermediate goods are negotiated, and there are only two downstream firms before merger, whereas in our paper, both prices and quantities are negotiated, and there are three downstream firms before merger. These differences result in the difference in the profitability of merger between Horn and Wolinsky (1988) and our paper. Moreover, in Horn and Wolinsky (1988), since there are only two downstream firms, the merger paradox does not occur, whereas our paper presents an example in which merger is not profitable in market transactions (the merger paradox), but it is profitable in negotiated transactions.

Chipty and Snyder (1999) investigate the situation where there are one supplier and multiple buyers and the supplier and each buyer simultaneously negotiate a quantity and price. Chipty and Snyder (1999) provide a condition for the merger of two buyers to increase their profits. This increase is due to the enhancement of their bargaining position. In Chipty and Snyder (1999), buyers' products are perfectly heterogeneous, and thus, the merger paradox does not occur, which is the main difference from our paper.

The remainder of this paper is organized as follows: Section 2 defines partially cooperative games and Nash solutions, Section 3 provides a sufficient condition for a Nash solution to exist, Section 4 shows that Nash solutions approximately coincide with the equilibrium outcomes of extensive form games, Section 5 presents a solution to the merger paradox as an application, and Section 6 concludes the paper.

2. Notations and definitions

For any sets I and X, let X^I be the set of families of elements in X indexed by I. For readability, we denote the term of family x at index i by x^i as well as x_i. If the index is a coalition or a member of a coalition, we use subscripts; if the index is a discount factor, a period or a proposer, we use superscripts (a coalition, a discount factor et cetera are defined below).

2.1. Partially cooperative games

Definition 1. A *partially cooperative game* is a quadruple $\left(N, \mathcal{C}, ((S_C, \bar{s}_C))_{C\in\mathcal{C}}, (u_i)_{i\in N}\right)$ such that N is a nonempty finite set; $\mathcal{C}$ is a cover of N, i.e., a set of nonempty subsets of N such that $\bigcup \mathcal{C} = N$[3]; for any $C \in \mathcal{C}$, $(S_C, \bar{s}_C)$ is a pointed set, i.e., $\bar{s}_C \in S_C$; for any $i \in N$, $u_i : \prod_{C\in\mathcal{C}} S_C \to \mathbb{R}$.

2 Manea (2011) investigate bargaining in networks, where each pair of linked nodes (players) potentially produces a surplus. In his model, at each period, only one pair of players negotiates. Thus, his model is not closely related to interdependent bargaining situations.

3 Some authors denote $\bigcup \mathcal{C}$ by $\bigcup_{C\in\mathcal{C}} C$.