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# Multi-period mean–variance fuzzy portfolio optimization model with transaction costs

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ABSTRACT

This paper examines the multi-period portfolio optimization problem with transaction costs and fuzzy variables to count for the uncertainty of future returns and liquidities on assets. The portfolio risk is quantified by using the variance of fuzzy returns. Two conflicting optimization objectives, namely, maximizing the terminal wealth and minimizing the cumulative risk of portfolios over the entire investment horizon, are taken into consideration. For solving the proposed model we introduce a new multi-objective evolutionary algorithm. Finally the performance of the proposed algorithm is compared with the NSGAII and MOEA/D with the assistance of real data from FTSE-100 in London.



## 1. Introduction

In the conventional Markowitz's Mean–Variance (M–V) framework (Markowitz, 1952), the return of a financial asset is represented by a random variable that follows the Gaussian distribution. The assumption of the normality distribution implies that the assets' returns are characterized only by their mean and variance. This assumption provides the basis of the mean–variance framework. However according to Doganoglu et al. (2007) assets' returns are not normally distributed and that implies that higher moments should be taken into consideration in the process of selecting the optimum portfolio.

Moreover, in an uncertain economic environment it is not possible to predict accurately the future returns and risks associated with the various financial assets, based exclusively on historical data. Fuzzy set theory can deal with uncertainty and vagueness which are usual features of any stock market. Fuzzy logic (Zadeh, 1978) can express uncertain knowledge making it suitable for representing the inherent uncertain nature of the portfolio optimization problem.

The traditional single period portfolio optimization models (Liagkouras and Metaxiotis, 2016a) only consider how to select the optimal portfolio at the beginning of investment period and imply to hold it until the end of the investment period. In practice, however, the investors evaluate and reallocate their wealth from period to period by taking into consideration the volatile market conditions. Thus, the investigation of the multi-period portfolio optimization problem with re-allocation, satisfies practical concerns and covers a gap in the available literature.

A number of different approaches have been proposed for handling the complexities of the multi-period portfolio optimization problem. In this study we classify the various studies into two distinct categories based on the assumptions about future returns. The group of studies that belong to the first category assumes that the returns on assets can be adequately described in terms of a probability distribution of a random variable. Some good examples of studies that belong to this category are listed below. In particular, Li and Ng (2000) considered the multi-period portfolio selection problem from a dynamic programming approach. Geyer et al. (2009) considered the multi-period investment problem with the assistance of a stochastic linear programming approach. Chen (2005) utilized CVaR as the measure of risk for solving the multi-period consumption and investment problem. Zhu et al. (2004) incorporated bankruptcy into the multi-period portfolio optimization model and formulated it as a bi-objective problem, namely mean and variance.

There is a growing number of studies that assert that it is not possible to predict future performance of assets based on historical data. According to these studies, the assumption of the normality distribution of returns on assets does not hold, due to the uncertainty and vagueness associated with risky assets. Fuzzy set theory is well suited to describe and treat imprecise and uncertain elements in the decision making process. The selection and re-balancing of the optimal portfolio clearly belongs to this category of problems. Below, we provide some typical examples of studies that investigate the selection of the optimum portfolio from a fuzzy logic perspective. In particular, Chen (2015), Gupta et al. (2013), Gupta et al. (2014), Huang (2011, 2012),

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Li et al. (2012), Liu and Zhang (2013) and Zhang et al. (2009) are some well-known examples of studies that consider the portfolio selection problem from a fuzzy logic perspective.

The aforementioned studies consider the single-period fuzzy portfolio optimization problem. However, over the last years, practical concerns urged scholars in the field to consider the multi-period fuzzy portfolio optimization problem. In particular, Sadjadi et al. (2011) introduced different rates for borrowing and lending into the multi-period portfolio optimization model. Bertsimas and Pachamanova (2008) handled the multi-period portfolio optimization problem with the assistance of a computationally efficient robust optimization approach. Zhang et al. (2012) proposed a mean semivariance-entropy model for fuzzy multi-period portfolio selection. Wang and Liu (2013) introduced fixed and proportional transaction costs into the multi-period mean–variance portfolio optimization model.

Neural networks have been used on a variety of problems and over the last years have been successfully applied to multi-objective problems. He et al. (2017) propose an adaptive neural network for controlling a marine vessel. The authors implement adaptive neural networks to approximate system's uncertainties. According to the authors under the proposed method the controller is able to achieve the desired constrained output. Finally, the authors provide numerical simulations to verify the feasibility of the proposed controller. In another study, He et al. (2016) propose an adaptive neural impedance control for a robotic manipulator. The authors approximate the system's uncertainties by introducing a radial basis function neural network controller. Finally, the authors provide simulations to verify the efficacy of the proposed controller. He et al. (2016) examine the tracking control problem for a robotic system. The authors propose an adaptive neural network control for handling system's uncertainties and disturbances. A barrier Lyapunov function is used to guarantee the uniform ultimate boundedness of the closed-loop system. The authors provide simulations to demonstrate the efficacy of the proposed control. He et al. (2017) propose a neural network controller for suppressing the vibration of a flexible robotic manipulator system. According to the authors the proposed neural network controller is able to compensate for the estimated deadzone effect and track the desired trajectory. The authors provide simulations to indicate the performance of the proposed neural network (NN) controller.

The multi-period portfolio optimization problem belongs to the category of multi-objective problems as two or more conflicting objectives (i.e. total wealth and cumulative risk) are optimized at any time (Liagkouras and Metaxiotis, 2014). The majority of studies in the field convert the multiobjective optimization problem to a single-objective optimization problem. Single-objective approaches such as goal programming or multicriteria decision-making methods convert all but one of the objective functions into constraints. Then, the formulated single-objective problem is solved multiple times, for different levels of the objective value, hoping to find each time a different Pareto optimal solution.

In this study in contrast to the majority of studies in the field we propose a multiobjective evolutionary algorithm (MOEA) to find multiple Pareto optimal solutions simultaneously (Liagkouras and Metaxiotis, 2015a, 2016b; Metaxiotis and Liagkouras, 2012). Also, for adding realism in the proposed model, the return and liquidity on assets are formed into trapezoidal fuzzy numbers, for expressing the vagueness and uncertainty of future returns and liquidities alike of the various financial assets. Moreover, in contrast to the majority of the studies in the field that consider the single-period portfolio optimization problem, in the present study we consider the multi-period portfolio optimization problem, where the wealth can be reallocated at the beginning of each period. Finally, in contrast to the majority of the studies in the field (Gupta et al., 2013; Liu et al., 2012, 2016; Mehlawat, 2016) that consider test instances that range between 10 and 20 stocks in the present study the examined test instance is considerably bigger, reaching the 92 assets, thus adding to the realism of the examined model.

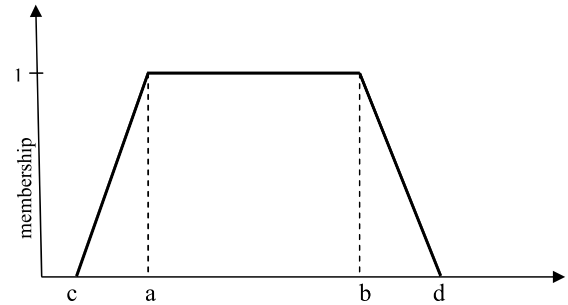


**Fig. 1.** The trapezoidal membership function $(a, b, c, d)$ for the returns on assets.

For solving the proposed model, we introduce a new multi-objective evolutionary algorithm, specially designed for handling the complexities of the multi-period mean–variance fuzzy portfolio optimization problem with transaction costs. The proposed algorithm is tested in comparison with the NSGAII and MOEA/D with the assistance of real data sets from the FTSE 100 index in London and obtains well-diversified portfolios for all examined cases.

The rest of the paper is organized as follows. In Section 2, we provide some introductory concepts and definitions. In Section 3, we present the multi-period mean–variance fuzzy portfolio optimization model with transaction costs. In Section 4, the proposed Multi-period Fuzzy Portfolio Optimization Algorithm (MFPOA) is presented. The test problems and the performance metrics are presented in Section 5. In Section 6, the simulations of MFPOA comparing with the NSGAII and MOEA/D are presented and the relevant results are analyzed. Finally, Section 7 concludes the paper and identifies possible research directions.

## 2. Introductory concepts and definitions

Financial markets are characterized by uncertainty which makes difficult the precise estimation of future returns associated with the various assets. Fuzzy logic is suitable for expressing the vagueness and uncertainty of future returns of the various financial assets. The return on assets can be formed into fuzzy values with the assistance of a membership function.

### *2.1. The expected value, variance and covariance of the trapezoidal fuzzy returns*

In this section we introduce some definitions which are needed for the rest of the paper. Fuzzy set theory is designed to allow the gradual assessment of the membership of the elements in relation to a set. The degree of membership of the various elements is performed with the assistance of a membership function $\mu_A(x) \to [0,1]$, which maps all elements in the interval between 0 and 1. Below, we will show how to construct a membership function for the returns on assets. A fuzzy number $\tilde{A}$ is called trapezoidal with core interval $[a, b]$, left width $c$ and right width $d$ if its membership function has the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{\alpha - x}{c}, & if\ a - c \le x \le a \\ 1, & if\ x \in [a, b], \\ 1 - \dfrac{x - b}{d}, & if\ b \le x \le b + d \\ 0, & if\ otherwise \end{cases} \tag{1}$$

and it can be represented as the notation $\tilde{A} = (a, b, c, d)$.

Since we assume that the future returns of the assets are trapezoidal fuzzy numbers, as shown in Fig. 1, we need to estimate the core interval and the left and right width of the fuzzy numbers. For calculating the trapezoidal fuzzy return rates of the proposed model, we use estimation method proposed by Vercher et al. (2007). According to Vercher et al. (2007) the historical returns are treated as sample and