Insurance: Mathematics and Economics 74 (2017) 31–45

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Optimal dividend strategies with time-inconsistent preferences and transaction costs in the Cramér–Lundberg model

Shumin Chen[a], Yan Zeng[b,*], Zhifeng Hao[c]

[a] School of Management, Guangdong University of Technology, Guangzhou 510006, PR China
[b] Lingnan (University) College, Sun Yat-sen University, Guangzhou 510275, PR China
[c] School of Mathematics, Foshan University, Foshan 528000, PR China

HIGHLIGHTS

- A dynamic time-inconsistent dividend problem with transaction costs is studied.
- The optimization problem is solved for both naive and sophisticated managers.
- Explicit optimal strategies and value functions are derived with mild conditions.
- Some theoretical and numerical analyses are presented to illustrate our results.
- Some examples and interesting phenomena are provided.

ARTICLE INFO

Article history:
Received October 2016
Received in revised form
February 2017
Accepted 22 February 2017
Available online 4 March 2017

Keywords:
Time-inconsistent preferences
Transaction costs
Hyperbolic discount rate
Optimal dividend strategy
Stochastic impulse control

ABSTRACT

This paper considers the optimal dividend strategies for an insurance company with transaction costs and time-inconsistent preferences. We assume that the company's surplus is modeled by a compound Poisson process and that the manager is either naive or sophisticated. We tackle the optimal dividend problem when the claim sizes belong to a certain class of distributions and the optimal dividend strategies are of the lump sum type. Our results indicate that a time-inconsistent manager tends to pay out dividends earlier and more frequently than a time-consistent manager, but with smaller dividend amounts. We also present the special case where claim sizes follow mixed exponential distribution to illustrate our results and to analyze the effect of time-inconsistency and transaction costs on the optimal dividend strategies.

1. Introduction

The development of mathematical tools in financial engineering and actuarial science has lead to an increase in research on optimal dividend strategies for insurance companies. (See Avanzi, 2009 and Albrecher and Thonhauser, 2009 for a review of this issue.) These studies use the classic Cramér–Lundberg (C–L) model to describe the surplus of insurers. Their aim is to maximize the expected net present value (ENPV) of dividend payments received by shareholders until the time of ruin, which is defined as the first time when the company's surplus becomes negative. As the ENPV is one of the most popular ways to measure the value of companies or investment projects (see Sethi et al., 1984), the optimal dividend problem has become a central issue in actuarial science and corporate finance research.

In the classic C–L framework, the optimal dividend problem was first solved by Gerber (1969) via a limit of an associated discrete-time problem, and later considered by Schmidli (2008), Azcue and Muler (2005, 2010), and so on. In these studies it is assumed that there are no transaction costs when dividends are paid out. However, Bai and Guo (2010), Thonhauser and Alberecher (2011) and Hunting and Paulsen (2013) show that small fixed transaction costs can have significant effect on the optimal value function and optimal dividend strategy. Bai and Guo (2010) study an optimal dividend problem for the C–L model and obtain analytical solutions when the claim sizes are exponentially distributed. Their results show that, in the presence of a fixed transaction cost, a so-called *lump sum strategy* characterized by a pair of non-negative parameters, (a, b), is optimal. That is, when the surplus is equal to or larger than b (called the *upper dividend barrier*), it is reduced to a ($<b$, called the *lower dividend barrier*) through a dividend payment. Thonhauser and Alberecher (2011) consider a similar problem,

* Corresponding author. Fax: +86 20 84114823.
E-mail addresses: chenshumin1@gmail.com (S. Chen), zengy36@mail.sysu.edu.cn (Y. Zeng), mazfhao@scut.edu.cn (Z. Hao).

http://dx.doi.org/10.1016/j.insmatheco.2017.02.009
0167-6687/

but assume that shareholders have power utility; they develop a numerical procedure to deal with the optimal dividend problem when claim sizes have general distributions. Recently, Hunting and Paulsen (2013) consider the optimal dividend problem for a general jump–diffusion model, and show that if claim sizes belong to a certain class of light-tailed distributions, the optimal dividend strategies are of the lump sum type.

In the aforementioned studies, it is assumed that dividend payments are discounted exponentially with a constant discount rate of time preferences. Such preferences are *time-consistent* in the sense that, at any time, the manager's preferences for dividend payments at an earlier date over a later date are the same. A manager with time-consistent preferences is termed a *time-consistent manager*. However, the experimental study performed by Thaler (1981) indicates that the standard assumption of time-consistent preferences (related to a constant discount rate) is unrealistic. Ainslie (1992) and Loewenstein and Prelec (1992) obtain the same result, and show that human beings are impatient when making choices in the short term but patient when making choices about long-term alternatives.[1] The effect of time-inconsistent preferences has been widely analyzed by Karp (2007), Grenadier and Wang (2007), and Marín-Solano and Navas (2010) in the context of planning economic growth, real options, and the classic optimal investment–consumption problem, respectively.

In this study, we revisit an optimal dividend problem for the C–L model to illustrate the effect of time-inconsistent preferences on an insurance company's dividend distributions. We also assume that dividend payments are subject to both fixed and proportional transaction costs. To reflect the empirical pattern of a declining discount rate, we follow Grenadier and Wang (2007), Harris and Laibson (2013), and Zou et al. (2014), and model managers' time-inconsistent preferences using a continuous-time quasi-hyperbolic discount function. Moreover, in line with standard research on time-inconsistent behavior, we formulate "the manager" as a sequence of autonomous selves who make decisions on successive decision horizons. At her decision horizon (her "present period"), each self formulates a dividend rule for the company by taking into account the conjectured decisions of her future selves (in her "future period"). We use the current self's vision of her future selves' preferences and the induced behaviors to define the manager as either naive or sophisticated, and consider the optimization problem for both cases. For a naive manager, the current self pays out dividends without realizing that her future selves will have different preferences. Without a mechanism to bind her future behaviors, the naive manager will deviate from an ex ante optimal plan in the future. Such strategies are called time-inconsistent strategies.[2] Obviously, a time-inconsistent strategy is not preferable in practice. To obtain a time-consistent plan, a manager must be sophisticated, i.e., must take into account the fact that her future selves will act according to their own preferences. In this case, the optimal dividend problem is formulated as an intra-personal game between successive players and is solved by looking for an intra-personal equilibrium strategy for the associated subgame with an infinite number of decision-makers.[3]

To solve the time-inconsistent manager's problem, we start by considering the time-consistent manager's problem as a benchmark, and then based on the results we consider the naive and sophisticated managers' problems. Inspired by Paulsen (2008), Bai et al. (2012), and Hunting and Paulsen (2013), we restrict ourselves to considering lump sum dividend strategies, and characterize the time-consistent, naive, and sophisticated managers' optimal strategies using three different systems of non-linear equations. Under certain mild conditions, we are able to explicitly determine the optimal value functions and the optimal strategies. Finally, we use the mixed exponential distribution as an example to illustrate our results and to analyze the effect of time-inconsistent preferences and transaction costs on managers' decision making.

We perform both theoretical and numerical analyses. First, we show that both time-inconsistent managers (especially the sophisticated manager) tend to pay out dividends earlier than the time-consistent manager and, to decrease the risk of bankruptcy, tend to pay out dividends more frequently and with smaller dividend amounts. Second, for both naive and sophisticated managers, our results show that the degree of impulsiveness in decisions to pay out dividends depends on a manager's degree of time-inconsistency. Managers with a larger degree of time-inconsistency tend to pay out dividends earlier and more frequently, but with smaller amounts each time. Finally, we observe that the fixed and proportional transaction costs have a reverse effect. When transaction costs become larger, all managers tend to pay out dividends later, less frequently, and with smaller amounts each time.

Recently, there are also literatures that involve the optimal dividend problems with time-inconsistent preferences. Using quasi-hyperbolic discounting function to depict the manager's time-inconsistent preferences, Chen et al. (2014) study the optimal dividend problem for a C–L model with diffusion, and Chen et al. (2016) study the optimal financing-dividend problem for a dual risk model, respectively. On the other line, Zhao et al. (2014) and Li et al. (2016) focus on the optimal dividend problem for a diffusion model and a dual risk model with a general non-exponential discounting function. It is worth noting that these papers only assume proportional transaction cost and do not consider the impact of fixed transaction cost.

The remainder of this paper is organized as follows. Section 2 introduces the surplus of the insurance company and the manager's time preferences. Section 3 provides some preliminary results, which are used to solve our optimization problem. Section 4 derives and analyzes optimal value functions and optimal dividend strategies for a time-consistent manager, naive manager, and sophisticated manager. Section 5 first considers the special case where the claim sizes are mixed-exponentially distributed, to show the effectiveness of the results presented in Section 4, and then provides several numerical examples to illustrate the effect of time-inconsistent preferences and transaction costs on a company's dividend strategies. Section 6 concludes the paper. Appendix gives the proofs of our results.

2. The model

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions, i.e., filtration $\{\mathcal{F}_t\}_{t\geq 0}$ is right-continuous and $\mathbb{P}$-complete. $\mathcal{F}_t$ represents the information available up to time t, and all of the decisions made at time t are based on this information. All of the stochastic processes introduced below are well-defined and adapted to this probability space.

2.1. Surplus process

Suppose that the uncontrolled surplus process $R = \{R_t\}_{t\geq 0}$ of an insurer is

1 For instance, Ekeland et al. (2012) point out that one might prefer to get two oranges in 21 days rather than one orange in 20 days, but might also prefer to get one orange right now rather than two oranges tomorrow. These time-inconsistent preferences are called "present-biased" preferences by O'Donoghue and Rabin (1999).

2 In addition to time-inconsistent preferences, there are many other conditions that lead to time-inconsistent strategies such as rank-dependent utility, probability weighting, etc. (see Hu et al., 2012, Björk and Murgoci, 2010, and He and Zhou, 2016).

3 The equilibrium framework is a standard approach for obtaining a consistent plan when facing a time-inconsistent decision-making problem, see Grenadier and Wang (2007), Harris and Laibson (2013), and Zou et al. (2014).