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Optimal execution with price impact under Cumulative Prospect Theory

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HIGHLIGHTS

- We propose an optimal execution model with transient price impact and permanent price impact.
- Traders' behaviors are described using Cumulative Prospect Theory.
- Dynamic reference points are introduced in the decision process.
- We obtain some properties of trading strategies under different market circumstances.

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ABSTRACT

Optimal execution of a stock (or portfolio) has been widely studied in academia and in practice over the past decade, and minimizing transaction costs is a critical point. However, few researchers consider the psychological factors for the traders. What are traders truly concerned with — buying low in the paper accounts or buying lower compared to others? We consider the optimal trading strategies in terms of the price impact and Cumulative Prospect Theory and identify some specific properties. Our analyses indicate that a large proportion of the execution volume is distributed at both ends of the transaction time. But the trader's optimal strategies may not be implemented at the same transaction size and speed in different market environments.

1. Introduction

For stock traders, a common problem is unwinding the large block orders of shares, which can comprise up to 20% of the daily traded volume of shares [1]. Orders of this size may induce significant price impacts and execution costs.

For example, if a trader wants to buy a large amount of a stock (or a portfolio), the traditional method is to submit the bid orders to the limit order book and use the liquidity to absorb the requested orders. However, this trading method may induce two possible effects. First, when the market does not have sufficient liquidity, market participants are not willing to make frequent trades (as they are sitting on the sidelines). The large amount that the trader wants to trade cannot be successfully executed; meanwhile, the transaction costs may be excessive due to the lack of liquidity. Second, if the market participants are both enthusiastic and sensitive to the price discovery process, there will be a large proportion of participants who chase trends. Thus, block trades are likely to follow and result in situations similar to those mentioned above.

The above discussion leads to introducing an important concept—price impact, meaning that the price changes with an incoming order [2,3]. Naturally, a purchase order may increase the price, and a sell order may induce the opposite effect. In

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the Kyle model [4], the price adjustment rule of the market maker must be linear for the total signed volume. Bertsimas, Lo and Hummel derive the dynamic optimal trading strategies that minimize the expected cost of trading a large block of equity over a fixed-time horizon [5,6]. Some researchers have found that the volume dependence of this impact is sublinear and is well described by power-law [7,8]. They show that the market impact cost per share is taken to be a power-law function of the trading rate with an arbitrary positive exponent [9–15].

However, the empirical data show that in a large variety of markets, the autocorrelation of the signs decays extremely slowly over at least several days, representing thousands of trades or more [16]. Therefore, the price impact may be meticulously divided into the transient price impact and permanent price impact. The transient price impact is also known as a *price shock*, and the permanent price impact is a comprehensive result of price shock and price decay. There are two basic concepts of price shock (that generally accept exponential price decay), one of which is linear price shock. Obizhaeva and Wang hold this opinion and propose a linear transient price impact model with exponential decay [17]. Another concept is the power law. Almgren, Thum, Hauptmann and Li postulate that the temporary impact function is a power-law type. Their analysis confirms the concavity of the function with a temporary cost exponent equal to 0.6 [18]. Alfonsi, Fruth and Schied also consider optimal execution strategies with nonlinear market impact and exponential resilience [1]. In particular, Zhou investigates the determinants of immediate price impact at the trade level [14,19]. In addition, many researches discuss the transient price impact and price decay in various ways [20–25].

Under the classic framework of finance, market participants are rational investors. However, the reality is that many investors do not have regular thinking patterns with regard to decision making. Tversky and Kahneman proposed the *Prospect Theory (PT)* with regard to decision making in uncertain situations [26], they developed an original version of this theory, which led to the *Cumulative Prospect Theory*. Their studies showed that people are more risk-seeking with small probability gains and large probability losses while being risk-averse with small probability losses and large probability gains, as a result, people overweight low probabilities and underweight moderate and high probabilities. Meanwhile, people may add their past efforts into their total costs, and therefore future expected wealth and past actions may affect their decision [27]. In recent years, behavioral factors and emotions have been given significant attention in portfolio studies. Based on the *(Cumulative) Prospect Theory*, many researchers have studied the behavioral optimal selection theory and its implications [28–30].

Although most previous studies on optimal trading strategies focus on price impact and execution costs, some researchers consider psychological factors. Our approach, therefore, is to combine the Obizhaeva and Wang framework with the utility of traders' mental accounting with taking price impact into account. Unlike most studies on behavior optimal execution, our work applies a series of reference points instead of a static reference point because decision makers may update their reference points upon receiving a sequence of information. These reference points are decided by the environment of the capital market. Withanawasama et al. show that when characterizing limit order prices, varying the offset distribution only produces different behavior when the reference price is the contra side best price [31]. Kőszegi and Rabin depict reference points as exogenous variables, not traditional static points [32]. They assume that a person's reference point is her rational expectations held in the recent past about outcomes. The "gain-loss utility" they used influences behavior when there is uncertainty. This method is consistent with our work because, from our point of view, a trader's object is not the traditional "buy low and sell high", but "buy lower and sell higher". He and Zhou find that in addition to the agent's loss aversion and evaluation period, his reference point also has a significant effect on optimal asset allocation [33]. The research on dynamic reference points and investors' performance has been subsequently extended by some scholars [34].

The rest of this paper is organized as follows. In Section 2, we describe the main building blocks for the model: the formation of price impact and impact decay, the trader's expected reference points and CPT (cumulative prospect theory) utility objective. In Section 3, we present an approximate solution for the optimal trade trajectory and relate it to the simulated market price, given the corresponding mental utility. We then conclude that compared to the strategy that buy lower than others, simply "buy low and sell high" does not actually offer the trader the optimal emotional experience.

2. Model formulation

We consider an institution trader's optimal execution problem. Suppose that an institution trader initially has a large amount of cash and that this trader wants to spend a fixed duration of time T buying one stock as a result. He will hold an inventory of X stocks until the terminal time T. We divide the total trading time into N individual and equal length time periods τ and define t_i as the initial point of every time period t_i, where $t_0 = 0$ and $t_N = T$. Let x_{t_i} denote the inventory amount for the total trade before t_i, where $x_{t_0} = 0$ and $x_{t_N} = X$.

Initially, this trader will invest at t_0. Suppose that the stock's initial price is P_0 and this trader's trade size after t_0 is u_{t_0}, where $u_{t_0} = x_{t_0}$. As a result, the stock price will be struck, inducing a transient price impact, and the price will increase to P_0^+ from P_0. Obizhaeva and Wang [17] proposed that this transient price impact can be expressed as

$$\int_{P_0}^{P_0^+} q\left(P\right) dP = x_{t_0}. \tag{1}$$

In the expression, $q\left(P\right)$ represents the density of the orders to buy at price P and the number of orders in a small price interval $[P_0, P_0 + dP]$ is $q\left(P\right) dP$. For simplicity, we assume that there is enough liquidity in the market, which means every