A simulation optimization based control policy for failure prone one-machine, two-product manufacturing systems

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Abstract

This paper presents the optimal flow control for a one-machine, two-product manufacturing system subject to random failures and repairs. The machine capacity process is assumed to be a finite state Markov chain. The problem is to choose the production rates so as to minimize the expected discounted cost of inventory/backlog over an infinite horizon. We first show that for constant demand rates and exponential failure and repair time distributions of the machine, the hedging point policy is optimal. Next, the hedging point policy is extended to non-exponential failure and repair time distributions models. The structure of the hedging point policy is parameterized by two factors representing the thresholds of involved products. With such a policy, simulation experiments are coupled with experimental design and response surface methodology to estimate the optimal control policy. Our results reveal that the hedging point policy is also applicable to a wide variety of complex problems (i.e. non-exponential failure and repair time distributions) where analytical solutions may not be easily obtained.
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1. Introduction

We are concerned with the problem of controlling the production rates of a stochastic one-machine, two-product manufacturing system in order to meet the demand rates facing the system at a minimum cost. The stochastic nature of the system is due to machines that are subject to breakdowns and repairs. The decision variables are the production rates of products. Our objective is to choose admissible production rates to minimize the inventory/backlog costs over an infinite horizon. Under some restrictive hypotheses (i.e. exponential distribution of breakdowns and repairs inter-arrival), the problem of

controlling a flexible manufacturing system or FMS with unreliable machines was formulated as a stochastic optimal control problem by Older and Suri (1980). Based on the method presented by Rishel (1975) and Kimemia and Gershwin (1983) showed that the optimal control policy of such a problem has a special structure called the hedging point. Investigation in the same direction gives rise to the analytical solution of the one-machine, one-product manufacturing system (Akella & Kumar, 1986). It remains difficult to obtain the optimal control of a large scale manufacturing system (i.e. involving at least two parts).

The optimality conditions of the above problems, based on stochastic models, are described by generalized Hamilton Jacobi Bellman (HJB) equations for optimal production policies. However, it is by now well-known that finding the analytical solutions of HJB equations is almost impossible except in a few very simple cases, such as in Akella and Kumar (1986). The crux of the hedging point policy is to maintain surplus levels or thresholds, corresponding to the optimal inventory levels, in order to compensate possible backlogs caused by random failures.

We observe that there is still no satisfying method for manufacturing systems modelled by non-Markov processes (i.e. non-exponential machine up and down times). We will first prove that, with exponential machine up and down times and constant demand rates, the combination of discrete event simulation modelling, experimental design and analytical control theory can provide a control policy that can be compared to the one given by the numerical approach (Kusner & Dupuis, 1992). Following this validation, we will extend our proposal to non-exponential machine up and down time situations.

The proposed approach consists of estimating the relationship between the incurred cost and the threshold stock levels considered herein as control factors. The hedging point policy, parameterized by these factors, is then used to conduct simulation experiments. For each configuration of input factors values, the simulation model is used to determine the related output or cost incurred. An input–output data is then generated through the simulation model. The experimental design is used to determine significant factors and/or their interactions and the response surface methodology is applied to the obtained input–output data vectors to estimate the cost function and the related optimum. Details on the combination of analytical approaches and simulation based statistical methods can be found in Abdulnour, Duddek, & Smith (1995), Gharbi and Kenne (2000) and Kenne and Gharbi (1999).

The remainder of the paper is organized as follows. Section 2 describes the statement of the optimal control problem. The description of the proposed approach and the simulation model are presented in section 3. Section 4 outlines the experimental design approach and response surface methodology. Section 5 is devoted to extensions of the proposal to non-markovian processes. Section 6 concludes the paper.

2. Problem statement

The system under study consists of one machine producing two part types. The operational mode of the machine can be described by a stochastic process $\zeta(t)$. Such a machine is available when it is operational ($\zeta(t) = 1$) and unavailable when it is under repair ($\zeta(t) = 2$). One can describe $\zeta(t)$ statistically with the following state probabilities:

$$P[\zeta(t + \delta t) = \beta | \zeta(t) = \alpha] = \begin{cases} \lambda_{\alpha\beta}\delta t + o(\delta t) & \text{if } \alpha \neq \beta \\ 1 + \lambda_{\alpha\beta}\delta t + o(\delta t) & \text{if } \alpha = \beta \end{cases} \qquad (1)$$