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An extended lattice model accounting for traffic jerk

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HIGHLIGHTS

- A one-dimensional flux difference lattice hydrodynamic traffic flow model is proposed by considering the traffic jerk effect.
- The qualitative properties of modified model are analyzed theoretically.
- Phase diagrams are presented for different parameters.
- The effect of traffic jerk is examined numerically.

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ABSTRACT

In this paper, a flux difference lattice hydrodynamics model is extended by considering the traffic jerk effect which comes due to vehicular motion of non-motor automobiles. The effect of traffic jerk has been examined through linear stability analysis and shown that it can significantly enlarge the unstable region on the phase diagram. To describe the phase transition of traffic flow, mKdV equation near the critical point is derived through nonlinear stability analysis. The theoretical findings have been verified using numerical simulation which confirms that the jerk parameter plays an important role in stabilizing the traffic jam efficiently in sensing the flux difference of leading sites.

1. Introduction

Due to increasing automobile and modernization construction, the traffic congestion has closely impacted on human's daily life such as pollution, traffic accident and global warning etc. Therefore, to alleviate the problem of traffic congestion, it has been highly necessary to study the reason behind the traffic congestion problem experimentally as well as theoretically. In order to understand the complex mechanism of traffic congestion and to control it, a large number of mathematical models have been proposed such as car-following models [1–9], continuum models [10–21] and lattice hydrodynamics models [22–31].

The lattice hydrodynamics model was firstly proposed by Nagatani [22] in order to analyze the properties of traffic density wave in term of kink antikink soliton. After that, LH model was universally studied by considering various factors such as effect of leading sites [32] and backward sites [33], effect of lane width and anticipation effect of potential lane changing [34–36], delayed-feedback control [37] and driver's delay effect on a uniform single as well as two-lane highway, etc. Furthermore, LH model has been extended on gradient highway [38] and to two-dimensional traffic networks [39–41].

In real traffic situations, the sudden braking and acceleration of vehicles can leads to traffic accidents, increase environmental pollution and large wastage of energy. There is a large number of non-motors vehicles on urban and country roads

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which influences the proceeding and following vehicles and it causes a sudden change in acceleration of vehicles which is called traffic jerk. In this direction, Ge et al. [42] investigated the affects of non-motor vehicles on traffic flow by introducing a new factor i.e. traffic jerk. Also, Liu et al. [43] presented a car-following by considering the influences of non-motor vehicles in term of traffic jerk and found that this effect enhances the traffic congestion. Furthermore, many researcher [44,45] also investigated the effect of traffic jerk at micro level and concluded that traffic jerk and full velocity difference relieved the traffic jam. On the other hand, Liu et al. [46] also studied the effect of traffic jerk in continuum models and investigated the influence of traffic jerk on roads flux. Furthermore, a modified continuum models [47] is also presented based on the full velocity difference by taking into account jerk effect and it is concluded that traffic jam suppressed efficiently. Therefore, jerk parameter affects the traffic flow efficiently and its play an important role in traffic flow modeling. But, up to now, no studies has been carried out to study the traffic jerk parameter in LH traffic flow model which is the intermediate approach of microscopic and macroscopic models. On the other hand, this approach is also easy to handle many important real factors of traffic flow.

A flux difference LH model is extended by involving the influences of traffic jerk on single-lane highway in Section 2. In Sections 3 and 4, linear as well as nonlinear analysis has been studied for the proposed model followed by numerical simulation in Section 5. Finally, conclusions are discussed in Section 6.

2. Lattice hydrodynamic model

In Ref. [22], Nagatani proposed a lattice hydrodynamic model to describe the traffic phenomena in term of density waves on smooth single-lane highway. The model is

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{1}$$

with the following flux evolution equation at site j

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_{j+1}) - \rho_j v_j] \tag{2}$$

where ρ_0 is the average density; a is the sensitivity of drivers; $V(.)$ is the optimal velocity function; ρ_j and v_j represent the density and velocity at site j at time t, respectively. The above model described the traffic congestion in term of density waves such as kink–antikink density waves and provide a platform to handle many real factor of traffic which affects the traffic flow significantly.

In this direction, Peng [48] developed a LH model by considering the driver anticipation effect by taking the idea that driver adjusts his running behavior by observing varying condition of flux at site $j + 1$. The flow evolution is

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_{j+1}) - \rho_j v_j] + a\kappa[\rho_{j+1} v_{j+1} - \rho_j v_j] \tag{3}$$

where κ is the anticipation coefficient of flux difference. From above model, it is found that anticipation coefficient leads to enhancement of free flow.

As we know, the sudden acceleration and deceleration of non-motor vehicles causes the complex traffic congestion in an urban city. In this direction, Ge et al. [42] developed a car-following traffic flow model by taking into account the effect of traffic jerk and described its affect on jamming transition. From the study of traffic jerk parameter at the micro-level, it is concluded that irregular motion of non-motor vehicles leads to enhancement of traffic jam and therefore, the traffic jerk parameter plays an important role in the theory of traffic flow But, no study has been done by considering the effect of traffic jerk and to overcome the traffic congestion induced by irregular motion of non-motor vehicles in lattice hydrodynamics model. On the other hand, it is also found that flux difference effect plays an important role in stabilizing the traffic jam. So, in order to understand the phenomenon of traffic congestion due to the motion of non-motor vehicles and to reduce it, we extend the flux difference LH model [48] by considering the role of traffic jerk. In the extended model, the continuity equation remains same while flow evolution is modified by adding the role of jerk parameter. The extended model is as follows:

$$\begin{aligned}\partial_t(\rho_j v_j) &= a[\rho_0 V(\rho_{j+1}) - \rho_j v_j] \\ &+ a\kappa[\rho_{j+1} v_{j+1} - \rho_j v_j] - a\lambda[\rho_j(t) v_j(t) - \rho_j(t-\tau) v_j(t-\tau)]\end{aligned} \tag{4}$$

where λ is the traffic jerk coefficient which correspond to vehicular motion of non-motor vehicles. The jerk parameter profile represents the temporal dynamics of the flow of vehicles and mostly, more number of non-motor vehicles on roads leads to traffic congestion and destabilize the flow.

By taking the difference form of Eqs. (1) and (4) and eliminating speed v_j, the density equation is obtained as

$$\begin{aligned}&\rho_j(t+2\tau) - \rho_j(t+\tau) + \tau\rho_0^2[V(\rho_{j+1}(t)) - V(\rho_j(t))] + \kappa[-\rho_{j+1}(t+\tau) + \rho_{j+1}(t) \\ &\quad + \rho_j(t+\tau) - \rho_j(t)] - \lambda[-\rho_j(t+\tau) + 2\rho_j(t) - \rho_j(t-\tau)] = 0.\end{aligned} \tag{5}$$

The optimal velocity function given by Nagatani [22] is adopted.

$$V(\rho) = \frac{V_{max}}{2}\left[\tanh\left(\frac{1}{\rho} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right)\right] \tag{6}$$

where V_{max} and ρ_c denote the maximal velocity and the safety critical density, respectively. The optimal velocity function is monotonically decreasing, has an upper bound and a turning point at $\rho = \rho_c = \rho_0$.